

7 Language, knowledge and authority in school mathematics

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For the academic linguist and educational sociologist, the language of mathematics fascinates. It is a language unlike any other: an amalgam of symbols and images with spoken and written English, which constructs forms of knowledge – ways of modelling the world – which are unlike any other and which have seemingly endless applications. In the mathematics classroom, language is used to construct knowledge and regulate access to that knowledge in ways totally different from those used in other pedagogical contexts.

For language educators, those with the job of sharing knowledge about language with teachers and students, the language of mathematics terrifies. Its very uniqueness means that one cannot bring in understandings about language from other fields of activity. There are no essays to write in mathematics, no great chunks of written prose in textbooks. Moreover, the whole field of mathematics education is so strongly insulated from other fields that the language educator often suffers from ‘impostor syndrome’, feeling out-of-place in the company of those who control the arcane and mysterious language of mathematics.

Like the language itself, *research* on the language of mathematics is itself very different from research on other areas of language education. Most descriptions of mathematical language are to be found in journals of mathematics education, not in journals of language education. To the mathematician, the research on mathematical language in language education journals frequently appears to be woefully inadequate in its understanding of mathematical knowledge. To the linguist, the research in mathematics journals seems horribly simplistic in the role it assigns to language in learning. The result of all this is that language educators and mathematicians rarely talk to one another.

This chapter takes steps towards synthesizing these two areas of concern: that the nature of mathematical knowledge be well understood and that the role of language in constructing and exchanging this knowledge be fully appreciated. The educational sociology developed by Basil Bernstein and his colleagues over the last thirty or so years (Bernstein 1977, 1990, 1996) and the systemic-functional linguistics (SFL) devel-

oped by Michael Halliday and his colleagues over a similar period (Eggins 1994; Halliday 1977, 1994; Halliday and Hasan 1985; Martin 1992; Mattheissen 1996) provide useful models for attempting this synthesis. Bernstein's sociology has as one of its central concerns the role of language in constructing knowledge through pedagogical discourse. Hallidayan linguistics is particularly sensitive to the dialectical relationship between forms of language and types of social context. Bernstein's sociology and Halliday's linguistics have, of course, been used together to synthesize the linguistic and sociological concerns of a number of other sites of pedagogical activity, including the primary school classroom (see Williams, Chapter 4, and Christie, Chapter 5 this volume), early schooling (Williams 1995) and mother/child interaction (Hasan 1996).

The first part of the chapter identifies and describes some of the distinctive features of language in the mathematics classroom, using descriptions from SFL. The texts and descriptions are taken from research conducted between 1992 and 1996 in socio-economically disadvantaged secondary schools in the inner suburbs of Sydney. The second part reflects on the nature of mathematical discourse as it is recontextualized in secondary schools using some of the insights of Bernstein's sociology.

Linguistic perspectives on mathematical discourse

There is much that can be, and has been, said about the linguistic forms to be found in mathematical texts and it is not intended to try and report on all aspects of mathematical language here. It is possible, however, to enumerate a number of linguistic features which make the kind of language used in mathematics classrooms distinctive. It should be noted that the discussion below is of limited scope and in no way should be considered a complete account of the language of mathematics. The discussion is limited in at least two ways. First, it is concerned mainly with how the language of mathematics is different from the language used to explore and construct other bodies of knowledge in the school context. Thus the meaning of 'distinctive' linguistic features is limited here to those linguistic features which make the language of mathematics different from the language of other school disciplines. There has been considerable research within SFL of the specific language features of a range of schools subjects which can be contrasted with the language of school mathematics. These include English (Cranny-Francis 1996; Martin 1996; Rothery 1994, 1996), geography (van Leeuwen and Humphrey 1996), history (Veel and Coffin 1996) and science (Martin 1993; Veel 1997, 1998). There are many other possible ways of describing the 'salience' or 'distinctiveness' of mathematical language in schools – how it is different from the language of university and research mathematics (i.e. the pressures put on language through pedagogical recontextualization), how it is different from 'everyday' language in non-pedagogical contexts (i.e. the pressures of institutional recontextualization), how it is different from

child language (i.e. ontological pressures), gender, etc. Second, the discussion of mathematical text is restricted to the use of spoken and written language in mathematical 'texts'. It has been convincingly argued (Kress and van Leeuwen 1996, Lemke 1998, McInnes and Murison 1992, O'Halloran 1996) that mathematical texts are 'multimodal', consisting of semiotically rich configurations of images, diagrams and physical activity as well as language. The meaning potential of multimodal mathematical text is thus far greater than that of any single element viewed in isolation, although Hasan (1996) has argued convincingly for the pre-eminent role of language in the great majority of multimodal texts, especially those used in schooling. Figure 7.1 illustrates a multimodal text in the written medium. Thus the claims being made here about mathematical language are necessarily only part of the overall picture, albeit an essential part.

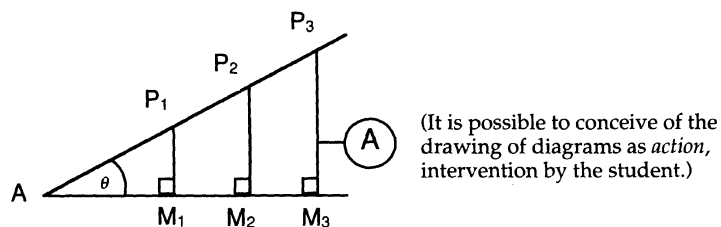
The distinctive linguistic features to be discussed here are:

- The predominance of teacher spoken language
- The predominance of distinctive patterns of spoken language interaction
- The technical fields of knowledge construed through spoken and written language
- The hierarchical ordering of mathematical concepts through language
- The gap between student use of mathematical language and teacher/textbook use of mathematical language

The predominance of teacher spoken language

Compared to other subject areas in the secondary school, mathematics relies extraordinarily on spoken language as the channel of communication for construing uncommonsense mathematical knowledge.¹ Whereas most other subject areas rely on an extensive canon of written prose (to be found in textbooks, encyclopedias and school libraries) to provide an impression of the stability and permanence of knowledge, this is noticeably absent in mathematics. Textbooks tend to be pastiches of repetitive activities and fragments of knowledge, and encyclopedia-style reference works are not present in the school context.

In a great number of mathematics classrooms there is a distinctive division of 'semiotic labour' between the spoken and the written modes. The written mode, through the use of the blackboard or (occasionally) the overhead projector, is mainly used for *symbolic* and *visual* construals of mathematical knowledge. The teacher's spoken language, on the other hand, provides a commentary on the visual and symbolic language being used on the blackboard. This commentary is often a vital aspect of the teaching situation, for it allows teachers to explain the meaning of the highly elaborated code of the symbolic and visual construals and to make links between students' (usually more 'everyday') construals of knowledge



Cosine Ratio

Since the triangles AP_1M_1 , AP_2M_2 and AP_3M_3 are similar.

These ratios are equal.

$$\frac{AM_1}{AP_1} = \frac{AM_2}{AP_2} = \frac{AM_3}{AP_3}$$

(S) (V)

This means that for the acute angle θ in a right triangle of any size the ratio

$$\frac{\text{adjacent side}}{\text{hypotenuse}}$$

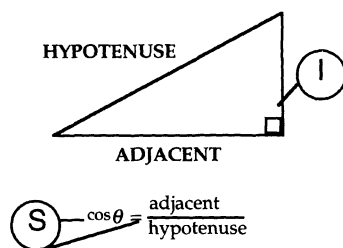
always has the same value.

This ratio is called the cosine ratio of angle θ .

In any right triangle the ratio

$$\frac{\text{adjacent side}}{\text{hypotenuse}}$$

for any acute angle is called the cosine ratio of that angle.



Key: (A) action (V) verbiage (I) image (S) symbol

Figure 7.1 Multimodal semiosis in written mathematical text (from McInnes and Murison 1992: 12)

and those officially recognized in the pedagogic field. It is spoken language which provides the link between the symbolic and visual representations for students, and is therefore a powerful agent in the learning process. Text 1 exemplifies the 'bridging' role played by spoken language in mathematics classrooms. The class is one of senior students (16 to 17), studying circle geometry and the teacher is explaining the meaning of radians as a unit of measurement.

Text 1

If we look at any circle, normally we take the unit circle to keep it simple. Any circle will do, if we draw in two radii, then we have the section of circumference between those two radii that is an arc length, and we have the angle subtended by that arc length at the centre of the circle, so we have that length we call it s , a section of the arc and the angle β at the centre of the circle. When we talk about a radian and we want to define what it means we want a radian to be a relationship between the length on the circumference, an arc length and the angle at the centre. Alright, so instead of talking about degrees and how much of a rotation we make, we're talking about this angle in terms of, if this is the angle how big will the arc length be, and for the unit circle we want it to be to correspond one to one, we want to say if this is one radian, then arc, that arc length is three. On any other circle, you take the arc length divided by what the radius is, if we're taking the radius to be one we don't have to worry about that, so it's a ratio between the arc length and the radius. Now, in our case, (sound of chalk) ... one radian (sound of chalk) ... subtends an arc (sound of chalk) ... of length one unit (sound of chalk) ... on the unit circle.

This reliance on the spoken mode has important social and linguistic consequences. It privileges particular forms of interaction in mathematics classrooms and it encourages 'strongly framed' teaching sequences. Over time, it has probably limited the kinds of knowledge written texts can construe and, therefore, what learners can achieve without access to a living, talking teacher. Text 2 illustrates what happens when a publisher attempts to provide through written language what a teacher will normally do in spoken language. The shift from the spoken to the written mode turns what is usually a fairly straightforward technique in algebra into an almost impenetrable theoretical piece. The text comes from a book which was designed for independent learning and aimed at adults who have little formal education in mathematics!

Text 2

Be an expert Study this definition

An axiom is a general statement which is accepted as true without proof.

Axiom 1: If equal quantities are added to equal quantities, the sums are equal.

This is called the addition *axiom*.

The addition axiom is used in solving an equation such as:

$$x-3=7$$

To solve this equation we must first find a value of x that will make the equation a true statement when substituted for the unknown quantity ' x '. Let us deal with the left hand member in such a way that all numerical terms are removed, leaving only the letter x . If we add $+3$ to -3 , the sum is 0 (zero). But, to maintain our mathematical balance, we must also add $+3$ to the right member.

$$\begin{array}{r} x-3=7 \\ +3=+3 \\ \hline x=10 \end{array}$$

By adding an equal value to both sides of the equation, we have changed it to an *equivalent* equation which is simpler in form and, in this problem, gives us the root.

In order to be certain, we must check the value which we found, by substituting it into the original equation.

$$\begin{array}{r} x-3=7 \\ 10-3=7 \\ \hline 7=7 \end{array}$$

(Herrick *et al.* 1962: 126)

The predominance of distinctive patterns of spoken language interaction

When one examines the spoken language of mathematics classrooms a second distinctive feature emerges. In those parts of the lesson where the spoken language consists of teacher-student dialogue, the interaction tends to be of a highly ritualized form, relying heavily on Input-Response-Evaluation (IRE) exchanges of the type first identified by Sinclair and Coulthard (1975). Extended sequences of IRE exchanges render the spoken interaction into a kind of catechism in which mathematical facts, previously introduced by the teacher, are elicited from students and then evaluated. The following extract from Text 3 (below) illustrates a typical IRE exchange:

Input: T: If I have something multiplied together, what do I do to get rid of it?
Response: Ps: Divide.
Evaluation: T: Divide.

As researchers note, the question asked by the teacher is not a 'genuine' one, in the sense that the teacher is not seeking information which she does not already know.² Thus the student's role is typically one of a catechistic parrot, rather than a genuine 'knower'.

Extending this analysis beyond IRE sequences, Martin (1992: 31-91),

in his examination of 'negotiation' in spoken interaction, distinguishes between 'primary' knowers and actors (encoded in the analysis as K1 or A1) and 'secondary' knowers and actors (encoded as K2, A2). Drawing on research by Ventola (1987), Martin further distinguishes other kinds of conversational moves in terms of whether they are 'delayed' (d), 'follow-up' (f), 'confirmation' (cf), reconfirmation (rcf) or 'clarification' (cl). In Martin's analysis, the IRE sequence is coded in the following way:

dK1: T: If I have something multiplied together, what do I do to get rid of it?
K2: Ps: Divide.
K1: T: Divide.

Thus for Martin it is in the 'evaluation' move of the IRE that the teacher declares herself to be the primary knower – the one with the answers. Even though students do provide information, the teacher's evaluation move relegates them to the role of 'secondary knowers'. Martin's analysis allows us to move beyond individual IRE sequences and see how conversational roles are ascribed in extended pieces of interaction. Text 3 is an excerpt from an extended piece of classroom interaction, coded using Martin's analysis.

Text 3 (from a Year 11 mathematics lesson)

Move	Analysis	Text
1.	dK1	T: What do I mean by opposite operations?
2.	cf	P: Pardon?
3.	rcf	T: What do I mean by opposite operations?
4.	K2	P: It's opposite.
5.	K1	T: Yeah,
6.	dK1	so if I have plus what do I do?
7.	K2	Ps: Minus.
8.	K1	T: Okay,
9.	K1	that's what I mean by opposite operations.
10.	K2	P: So you're going to have to take ...
11.	dK1	T: If I have something multiplied together, what do I do to get rid of it?
12.	K2	Ps: Divide.
13.	K1	T: Divide. All right,
14.	K1	so that's what I mean by opposite operations.
15.	K2	P: Are you going to put S equals a certain number and D equals a certain number?
16.	K1	T: No, I'm not going to substitute yet.
17.	K1	What I'm going to do is ...
18.	dK1	All right, Jim you tell me what the subject of that formula is?
19.	cf	P: Subject?
20.	rcf	T: Mmmm.

21.	dK1		What's the subject? Which pronominal is the subject?
22.	K2	P:	S
23.	K1	T:	S

In this extract we see that the teacher maintains the primary knower (K1) role in 14 of the 23 moves. Students maintain the secondary knower (K2) role in 8 of the 23 moves. Whenever the students do ask 'genuine questions' (i.e. where their K2 move is not preceded by a dK1 move by the teacher), the teacher either ignores the question (move 10) or very quickly ends the exchange and moves on to another (moves 15–17).

The long-term result of the unvaried use of interaction patterns such as that in Text 3 is that students are rarely given the opportunity to occupy the role of the 'knowers' or 'producers' of mathematical knowledge. They do not get the chance to construct extended and grammatically complex text (such as Text 1) in which generalized mathematical ideas are combined with measurable quantities in order to spin a rich web of mathematical knowledge. Not only does this interaction pattern accentuate the power difference between teacher and student, it creates serious problems for many students when they are placed in situations, such as written exams, where they *do* have to produce mathematical text independently. Unfortunately for students, it is precisely in these situations, and not in spoken classroom interaction, that we tend to assess them.

How have these catechistic kinds of spoken interaction in mathematics come about? In part they are a result of general traditions of classroom teaching, a vestige of more overtly authoritarian relationships between teachers and students. In part they arise from the heavy reliance on the spoken mode for construing knowledge. Careful control must be maintained over the spoken interaction in order for the desired kinds of meanings to be construed, for there can be no recourse to canonical written texts. In part they are due to the 'strongly classified' nature of mathematical knowledge itself (see below), where facts and ideas are seen as discrete units of knowledge.

The technical fields of knowledge construed through spoken and written language

As in many areas of technical knowledge, language is used in mathematics to construe systematically organized, technical bodies of knowledge. Many people will readily recognize a distinctive **technical lexis** in mathematics, but there are also a number of other, mainly grammatical, devices through which knowledge is construed. These include **grammatical metaphor**, **relational clauses** and a very particular use of the resources of the **nominal group**. We will examine each of these briefly.

TECHNICAL LEXIS

As Halliday (1977: 195–6) notes, technical lexis in mathematics consists both of items that have uniquely technical meaning and the re-use of non-

technical items as technical lexis. Items with a uniquely technical meaning are usually latinate (*parallel, denominator, bisect, quadrilateral*). Because of their unique technical meaning these items cause little confusion for students (although they may still be difficult to learn). The re-use of non-technical items as technical lexis occurs particularly in items which describe mathematical processes (*find, simplify, integrate, get, reduce, power, average*) and require students to distinguish between the meaning of these items in mathematical fields of activity and their meaning in non-mathematical fields.³ Most mathematics teachers are aware of the potential difficulties presented by technical lexis and both introduce and reinforce this lexis with great care.

GRAMMATICAL METAPHOR

Grammatical metaphor refers to the re-configuration of meanings in text, where more 'congruent' linguistic representations of the world (events represented through verbs/verb groups; sequences and logical relationships represented through conjunctions; qualities through adjectives) are recast for the purposes of creating new knowledge, placing objects and events in relationships to one another that are not necessarily congruent with our everyday experience of the world (events and qualities represented as nouns; logical relationships as verbs). The use of grammatical metaphor in science to create chains of causality and technical categories has been explored in considerable detail by Halliday (Halliday 1993b, 1998) and the reader is referred to Halliday's work for further discussion of the formal properties and functions of grammatical metaphor.

Grammatical metaphor is certainly a prominent linguistic feature of mathematical text, as it is for scientific discourse. The following example illustrates grammatical metaphor in mathematics:

Which of the following is the best *estimate* for the *weight* of a hen's egg?
(New South Wales Board of Studies, 1990: 3)

In this example two 'virtual entities' are created through grammatical metaphor: *estimate* (the result of the process of estimating) and *weight* (a measure of how much something weighs). Both terms are grammatically metaphorical because they have more congruent forms in the verbs 'to estimate' and 'to weigh'. The usefulness of grammatical metaphor for mathematical discourse comes not simply from the creation of two previously non-existent entities, but because, once created, words such as 'estimate' and 'weight' can then be put into new relationships with one another, and with other elements, through the grammar of the clause. Hence we are able to construe a nominal group (see below) 'the best estimate for the weight of a hen's egg' which is itself part of a larger structure, the clause 'Which of the following is the best estimate for the weight of a hen's egg?'

Although grammatical metaphor is a feature of many formally organized bodies of knowledge, there are some particular functions of grammatical metaphor which are emphasized in mathematics.⁴ One of these is the creation of **quantifiable entities** for the purposes of calculation. Once an event or a quality has been turned into a 'thing', a noun, it can generally be counted. Consider, for example, the idea of 'change' in mathematics. Represented congruently, as a verb (to change), there are limited resources for describing change in a mathematically salient fashion:

it changes a lot
it changes often
it changes every two hours

Once we have re-constructed the event as a *thing* (the change), it becomes possible to quantify the amount of change:

25% change
50% change
70% change

Moreover, the newly created entity can be combined with other entities (actual or virtual) to realize new meanings:

rate of change
increasing rate of change
different rate of change between men and women
the change differential according to gender

Some areas of mathematics, such as calculus, would be literally unthinkable without grammatical metaphor.

Another characteristic use of grammatical metaphor in mathematics is **the reification of mathematical activities as topic areas, or concepts**. Thus the activity of 'multiplying' becomes the concept of 'multiplication'; 'adding up' becomes 'addition'. There is a substantial difference in meaning between the congruent representation of an activity and the reified naming of the concept or topic. It is generally thought, for example, that a student is able to multiply numbers without necessarily understanding the generalized concept of multiplication, and that when a student understands the concept of multiplication as well as the operation there is a qualitative change in the student's learning. Not surprisingly, this use of grammatical metaphor is particularly prevalent in mathematics education, where there is a need to distinguish between 'operational facility' and 'conceptual understanding'.

RELATIONAL CLAUSES

There is a tendency in mathematical language to exploit the meaning potential of relational clause types in English (see Halliday 1994; Chapter 5 for a discussion of clause types). Relational clauses are also a feature of scientific language, as described by Martin (1993), Wignell (1998) and Halliday (1993a). Here are some analysed examples of relational clauses from mathematics:

Relational: Attributive (non-reversible: x is a type of y ; x belongs to group y)

<i>A square</i> Carrier	<i>is</i> Process: Relational: Attributive	<i>a quadrilateral.</i> Attribute
<i>Three and four</i> Carrier	<i>are</i> Process: Relational: Attributive	<i>factors of twelve.</i> Attribute

Relational: Identifying (reversible: x is equal to y ; x stands for y)

<i>A prime number</i> Token/Identifier	<i>is</i> Process: Relational: Identifying	<i>a number which can only be divided by one and itself.</i> Value/Identified
<i>The mean, or average, score</i> Token/Identifier	<i>is</i> Process: Relational: Identifying	<i>the sum of the scores divided by the number of scores.</i> Value/Identified

In mathematics the function of **attributive clauses** appears to be very similar to that in science: to classify objects and events according to the technical taxonomies of the field. In doing so these clauses render explicit to students the organization of uncommonsense knowledge in mathematics and play an important role in apprenticing students into mathematical knowledge. In school textbooks, clauses such as those above are frequently placed in a box, or shown in large or coloured fonts. This visual prominence further underlines their significance.

Identifying clauses appear to function in a number of ways. Most obviously, they are used to introduce a technical term and to negotiate between technical and less-technical construals of knowledge, providing a bridge between 'what students (are assumed to) know' and 'what is to be learned'. In a clause such as 'The mean, or average, score is the sum of the scores divided by the number of scores', the Token, *the mean, or*

average, score, is the more technical term being introduced. In order to function in the learning context it must be assumed that the less technical Value, *the sum of the scores divided by the number of scores*, can be readily understood by students.

A second and vital function of identifying clauses is that they provide a nexus between linguistic and symbolic representation in mathematics. In an identifying clause the Process (often the verb *to be*) **is the linguistic equivalent of the equals sign** in algebraic representation. The identifying clause 'The mean, or average, score is the sum of the scores divided by the number of scores', for example, parallels the algebraic formula: $\bar{x} = \sum \frac{x}{n}$. In discussing and preparing students for algebraic representation teachers will often systematically manipulate spoken language until an identifying clause is reached. This provides the stepping-off point for continued work in the symbolic mode. Text 1, discussed earlier, contains just such a build-up to a relational identifying clause (shown in bold):

Alright, so instead of talking about degrees and how much of a rotation we make, we're talking about this angle in terms of, if this is the angle how big will the arc length be, and for the unit circle we want it to be to correspond one to one, we want to say if this is one radian then arc that arc length is three. On any other circle, you take the arc length divided by what the radius is, if we're taking the radius to be one we don't have to worry about that, so **it's a ratio between the arc length and the radius**.

A third, and less laudable, role for identifying processes in mathematics is that **they allow for the construction of multiple-choice questions**. In order to construct a multiple-choice question it is necessary to construe mathematical knowledge as a relationship of equivalence. The following multiple-choice questions, which come from the 1990 New South Wales *Year 10 General Mathematics Reference Test*, all construe knowledge as equivalence relationships. Identifying processes were used for at least 70 per cent of the multiple-choice questions in this test. The question 'stems' only are shown, and the Relational Identifying Processes are shown in italics.

On this scale, what *is* the length of the pencil?
 Written as a fraction, 0.03 *is equivalent to*.
 Which of the following *is* the best estimate for the weight of a hen's egg?
 The shape of this nesting box *could be described as*.
 The greatest increase in population *was*.
 When cut out and folded along the dotted lines, which shape *will not form* a cube?

The predominance of multiple-choice questions in public examinations has rightly been criticized by educators because they present mathematical understanding as atomistic fragments of knowledge, emphasizing the correctness of the response rather than the process used to achieve the response. From a linguistic viewpoint we can also see how multiple choice questions put pressure on language to represent knowledge as

equivalence relationships (*x is equal to y*). As well as using Relational Identifying Processes the formation of multiple-choice questions also requires the frequent use of grammatical metaphor and complex nominal groups (see below) in order to construe knowledge as 'correspondences between things'. The following example, examined earlier, shows this:

Which of the following	is	the best estimate	for the weight	of a hen's egg?
		<i>metaphorical entity</i>	<i>metaphorical entity</i>	
		Nominal Group		
Token		Value		

A question such as this is a long way removed from testing the *practical skill* of estimation, as examiners might claim it does. Rather, it is asking students to identify a linguistically construed relationship of equivalence. The question necessarily tests students' language skills as much as it does their mathematical understanding.

NOMINAL GROUP

In any uncommonsense discourse the resources of the Nominal Group are used to elaborate the meaning of a single entity by describing qualities of the entity, showing how it relates to other entities or by qualifying or restricting the range of meaning of the entity. A range of grammatical resources function to realize these meanings in the nominal group (Halliday 1994: 180-96). Although mathematical language exploits the full potential of the nominal group, three resources, **Pre-numerative**, **Classifiers** and **Qualifiers** play a particularly prominent role. The following example shows these three resources at work:

the volume of *a* *rectangular* *prism* *with sides 8, 10 and 12cm*
Pre-numerative **Deictic** **Classifier** **Thing** **Qualifier**

In mathematics **Pre-numerative**, phrases which precede the Deictic element of the Nominal group, function to select an abstract, but quantifiable, mathematical attribute with which to describe the main entity, the 'Thing' (The Deictic functions to indicate if the Thing is specific – *the, this, that* etc – or non-specific – *any, some, a*, etc.). Very often the Pre-numerative has the effect of endowing an everyday entity with a *mathematical attribute* (e.g. *The length of a pencil*). This attribute is often a kind of measurement (length, area, volume, temperature etc.) or a technical term for a numerical relationship (e.g. *the first derivative of the expression*). There is often a close link between a Pre-numerative and a previously-introduced mathematical formula. Thus when a student reads the instruction 'Find the area of a circle of diameter 12cm', the Pre-numerative *area of* needs to be linked to the formula $a = \pi r^2$ in order for the student to proceed.

Classifiers are used in mathematics (as they are in other disciplines) to realize taxonomic relations of type/sub-type between entities (i.e. the relationship between the Classifier and the Thing in the nominal group). Here are some examples:

Rectangular *prism* (general category: Prism, sub-type: rectangular)
Classifier **Thing**

Prime *number* (general category: number, sub-type: prime).
Classifier **Thing**

Classifiers are particularly evident in the areas of number and geometry, where elaborate multi-layered systems of classification exist.

Qualifiers function to restrict the range of meaning of the nominal group. Very frequently in mathematics qualifiers provide numerical specifications. Thus the Qualifier renders the Thing a **specifiable entity**, a most important feature in mathematics.

Table 7.1 summarizes the chief functions of the Nominal Group in mathematics.

Table 7.1 Resources of the Nominal Group in mathematics

Element	Example	Function
<i>Pre-numerative</i>	<i>the volume of</i>	Selects a quantifiable mathematical attribute with which to describe the Thing
	<i>a</i>	
<i>Classifier</i>	<i>rectangular</i>	Sub-classifies Thing into a taxonomically ordered grouping
<i>Thing</i>	<i>prism</i>	Entity which is being described
<i>Qualifier</i>	<i>with sides 8, 10 and 12cm</i>	Restricts the range of meaning of the Thing; provides specifiable attributes of the Thing

THE HIERARCHICAL ORDERING OF MATHEMATICAL CONCEPTS THROUGH LANGUAGE

A striking feature of mathematical language is the way it builds up hierarchies of technicality, with the construal of technicality at one level dependent upon the construal of technicality at a previous level. Each successive level of technicality takes the language user one step further away from any 'congruent' or 'everyday' construal of meaning. Equally striking is the *rapidity* with which this process occurs in mathematics education. Students are very quickly apprenticed into the technicality of mathematics, and this initial technicality is used as the basis for further levels of technicality. Any everyday or congruent representations very quickly give way to interlocking sets of technical knowledge.⁵ As a new level of technicality is introduced, subordinate and more congruent levels of technicality are disposed of. The following hierarchy shows this:

First level	length = how long something is width = how far across something is height = how far off the ground something is
Second level	area = length x breadth
Third level	volume = area x height

At the first level of technicality, the technical term is equated to a relatively everyday construal through a relational clause (*x is equivalent to y*). At the second level, the technical terms generated at the previous level (*length, width*) are combined to construe a new technical term, and the everyday construals (*how long something is, how far across something is*) are disposed of. At the third level, technical terms generated at the first and second levels (*height, area*) are combined to construe a new technical term, and technical terms construed at the second level (*length, breadth*) are disposed of. The technical term *volume* is thus at least two steps away from an everyday construal, and is very difficult to conceive of in everyday terms. This hierarchy (length–area–volume) is a relatively simple example. At more advanced levels of mathematics, many more hierarchical layers of technicality are added and any notion of an 'everyday' construal is literally unthinkable. Consider, for example, the many levels of technicality that have been built up to construe the following statement:

If p is positive at point P on a curve, then the tangent is positive at that point and its function is said to be an *increasing function* at P .

(Jones and Couchman 1981: 232)

THE GAP BETWEEN STUDENT AND TEACHER/TEXTBOOK USE OF MATHEMATICAL LANGUAGE

Given the considerable technicality of mathematical language and the rapidity with which it is built up, it is not surprising to find that there are significant differences between teachers/textbooks and students in the way they employ language in the classroom. Text 4, in which senior secondary students discuss the solution to a relatively simple perimeter problem with one another, will be used to contrast student language both with the 'official' language of teachers, as exemplified by Text 1, and with the written language of multiple choice questions, such as those discussed above.

Text 4

F = Female M = Male

Segments where students are reading from question are shown in italics.

- M1: You read the next one. Now.
 F: All right. *A five-metre length of fencing timber costs \$8.00 and fence posts cost \$5.00 each. If 9 metres of timber are needed to fence a triangular paddock and a fence post is needed for each metre of fence, explain whether the fence timber or the fence post will cost more and why?*
 M1: Yeah. This is a current problem. *A five-metre length of fencing timber costs \$8.*
 F: I think we could best read it to ourselves.
 M1: Yeah. Yeah. Go.
 (Period of silence)
 M1: Is it worth it trying to link this up? (Mmmm) Three there, three there and three there.
 F: Yeah. Two more.
 M2: *A five-metre length of fencing timber costs \$8.*
 F: Mmmm. One of these costs \$8 and the other one. One post costs \$5.
 M1: No. You've gotta work out the triangle, how many posts you need.
 F: Yeah . . . wouldn't you wanna find . . . 9 posts.
 M1: Yeah, you need how many posts.
 F: Wouldn't you want the perimeter, 'cause you're putting on a fence and –
 M1: Yeah.
 M2: All right. Ugh. First the um the per perimeter of the ah fence is 9 lengths. Right.
 F: Yeah.*
 M1: Mmmm.*
 M2: Is given. And . . .
 M1: Each length's . . .
 M2: But they don't give you (Mmm) the length of the triangle. It says 9 lengths. What is lengths?
 M1: Each length is 5 metres.
 M2: Each length is 5 metres. Okay, fair enough.
 M1: So the perimeter is 45.
 M2: Uh. Okay, so if there's 9 lengths, right . . .
 F: Yeah.*

- M1: Yeah.*
 M1: So the perimeter is 45. Okay.
 M2: 9 lengths.
 M1: Equals 45.
 M2: 1, 2, 3, 4, 5, 6, 7, 8, 9.
 M1: So you've got 45, divide that by 3.
 M2: So you've got 1,2,3 . . .
 F: Triangle. You've got three on each side.
 M2: Okay, so you've got 9 lengths of ummm . . .
 F: 5 metres.
 M2: 9 lengths of 5 metres.
 F: Which is where you get 45.
 M2: So 9 times 5 which is \$45. The cost of the . . .
 F: Metres.
 M1: No. 45 metres. The cost is \$8.
 F: Yeah the cost is \$8 . . . You get 45 metres.
 M1: Yeah.
 F: And if fencing posts, one post is one metre . . .
 M2: So the cost of um of 45 metres would be 5 times 45, right.
 M1: Yeah.
 M2: So the cost of um of 45 metres would be 5 times 45, right.
 M1: Yeah.
 M2: Which is \$225. That is for the . . . That's it.
 F: The timber.
 M2: Just the timber. Okay.
 M1: Yeah.*
 F: Yeah.*
 M2: Ah, the post would needed would be 9 plus 1 because there would be more posts and it would be . . . no wait, see how many posts around . . . 1, 2 3, 4, 5, 6, 7 . . .
 F: No.
 M2: See first you would draw a triangle (laughs) to see how many posts, see, see how many posts . . .
 F: No, no. Because it says that one post is one metre long.
 M1: Yeah one post, one post is . . .
 M2: Oh now.
 F: Yeah, you see. Read it Noel.
 M2: One post right one . . .
 F: . . . is one metre..
 M1: Is that the fence
 F: The thing is that a fence post is needed . . .
 M2: For each metre of fence . . . Okay . . . for each metre of fence.
 F: You've got 45 fence?
 M1: No.
 F: Or you've got 45 thing, er, 45 post . . .
 M2: But if it's an enclosed area, right, the number of um posts would be the same . . .
 M1: Yeah 'cause it says post.
 M2: as the number of metres, right?
 M1: Yep.*

- F: Yeah.*
M2: So that should be 45 posts.
M1: Posts. Yeah
M2: Okay, and since each post cost ...
F: ... cost \$5
M2: ... costs \$5 ...
F: Times by 5.
M2: It would be the same 225 bucks.
F: ... and so ... \$5 ...
M2: Oops, sorry.
M1: Wrong.
M2: It's wrong, wrong, wrong, wrong. 5 metres cost \$8.
F: Couldn't trust you.
M2: Five-metre lengths fencing costs \$8.
M1: \$8. That's that'd be 45 times \$8 ...
F: Yeah ... Yeah
M2: So the cost of the fencing ...
M1: ... fencing
F: Timber.
M2: Would be 8 times 45, not 5 times 45.
M1: Exactly..
F: Mmm
M2: So would be ... what ... 300, er, 360 ...
M1: 360, yeah.
F: 360.
M2: So 45.
M1: Thanks for supporting me on that.
M2: That' all right.
F: What are you trying to say?
M1: 47, 45 posts.
M2: So the posts would cost *\$225
F: *\$225. So it's cheaper *by using the post.

Because it allows for students to take on a range of roles in constructing and manipulating knowledge, it is easy to see why many teachers are keen to introduce group discussion into mathematics classrooms. Compared to the catechistic nature of teacher–student interaction, such as that in Text 3, student–student interaction provides the possibility for students to take on the role of ‘primary knower’ (K1) and ‘primary actor’ more frequently. Moreover, in student–student interaction these roles can be moved around more easily and students who are less willing and/or able to take on these roles can be coaxed into doing so. There is clear evidence in Text 4, for example, that initially hesitant students, such as M1, progressively contribute more and more to the group discussion as it unfolds. This is far less likely to occur with teacher–student interaction where there are twenty or more students in the classroom.

The chief issue which arises from Text 4 comes not from differences in interaction patterns, however, but from differences in the linguistic quality of the utterances. Compared to teacher talk and to textbooks, the stu-

dents' talk is far more like everyday spoken language. It does not exploit the meaning potential of grammatical resources such as the Nominal Group and Relational Process to construe technical knowledge to nearly as great an extent as the teacher talk or the test questions. Table 7.2 compares the student interaction with the teacher talk of Text 1 and the written mathematical language of multiple choice questions.

Table 7.2 Comparison of three grammatical features of student talk, teacher talk and exam questions

	<i>Student group interaction (Text 4)</i>	<i>Teacher talk (Text 1)</i>	<i>Multiple-choice test questions†</i>
Lexical density (i.e. average number of 'content words' per clause)	1.66	4.36	4.54
Ratio of relational processes to non- relational processes	1:1.05	1.13:1	2.5:1
Ratio of long Nominal Groups to short Nominal Groups*	1:2.86	1:1.06	1.22:1

† The first 10 multiple-choice questions from the *1990 Year 10 General Mathematics Reference Test*

* A short Nominal Group is one consisting of Deictic/Numerative + Thing or less

In terms of *lexical density*, the average number of content words, or lexical items, per clause (Halliday and Hasan 1985: 61–72), there is roughly one-third the number of lexical items per clause in the student talk as in either the teacher talk or the Reference Test questions. Although the *topic* of the students talk is mathematical, the quality of their talk insofar as it is measurable by lexical density is about the same as everyday conversation (Halliday 1985: 65). The greater lexical density of the other texts allows for the ‘packing-in’ of meanings into grammatical relationships in a way that is qualitatively different from everyday talk, and thus assists the construal of technical mathematical meanings. This is further confirmed by the comparison of **Nominal Group** structure and **Process type** across the text. In terms of the nominal group, we can see that there is a far greater proportion of short nominal groups in the student text, meaning that this text does not exploit the potential of the nominal group to select attributes, sub-classify and qualify nearly so much as the teacher talk or the test questions (clearly this differing exploitation of the Nomi-

nal Group is what gives rise to the differing lexical densities of the three texts). In terms of process type, the test questions appear to concern themselves far more with construing relationships between entities (equivalence, group membership, attributes etc.) than with describing events. In the teacher talk and the student talk, on the other hand, there is a fairly even balance between representing 'going-on' (actions, thoughts, speech) and representing relationships. Figure 7.2 sets out the differences between the three texts in graph form.

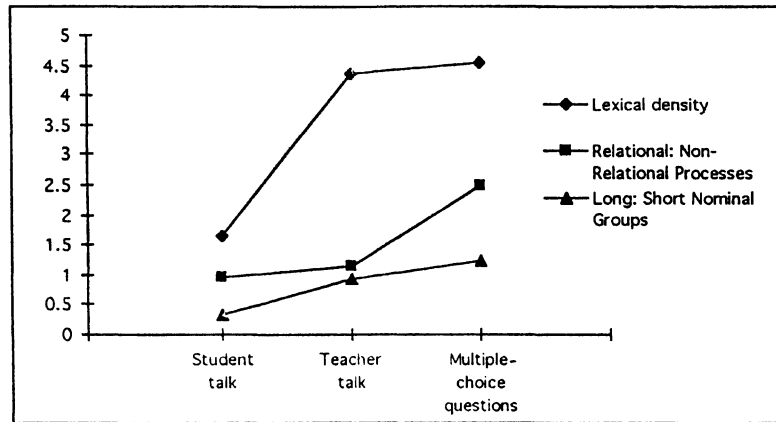


Figure 7.2 Trends in lexicogrammatical features across texts

In using SFL to analyse the language of mathematics in this chapter I have tried to provide explicit visible evidence with which to identify and explore a number of important issues in mathematics education. As the analysis shows, there are some clear, if not surprising, differences between expert and learner use of language in mathematics classrooms. It stands to reason that if students are to become competent at independently construing mathematical meanings, then increased control over the language of mathematics is one of the ways they can achieve this. Moreover, the linguistic analysis makes clear what could be the areas of focus for any potential language-based intervention in mathematics classrooms, and renders visible what might count as criteria of the success (or otherwise) of these interventions. However, the analysis also leaves open several important sociological questions. What is the nature of mathematical knowledge? Are the power relationships between experts and learners, so obvious in the language, a product of the nature of knowledge itself or of the way it is transmitted in the classroom, or both? If one is to 'democratize' mathematics education what kinds of issues have to be considered? To seek answers to these questions we need to apply sociological under-

standings to our linguistic analyses. The second part of this chapter attempts to do this.

Sociological perspectives on the language of mathematics

For many years now considerable attention has been paid to the way knowledge is construed in the mathematics classroom, and how that knowledge is perceived by students (Dossey 1992). In particular educators are concerned with the isolated, atomistic nature of mathematical knowledge as it is recontextualized in classrooms, and the perception by many students that mathematics comprises nothing more than a 'necessary set of rules and procedures to be learned by rote' (Crawford *et al.* 1994: 331). In a direct reaction to these perceptions, many maths educators stress the interconnectedness of mathematical knowledge:

We believe that the notion of connected representations of knowledge will continue to provide a useful way to think about understanding mathematics, for several reasons. Firstly, it provides a level of analysis that makes contact with the theoretical cognitive issues and practical educational issues . . . Second, it generates a coherent theoretical framework for connecting a variety of issues in mathematics teaching and learning, both past and present . . . Third, it suggests interpretations of students' learning that help to explain their successes and failures, both in and out of school.

(Hiebert and Carpenter 1992: 67)

It is this interconnectedness, Hiebert and Carpenter go on to say (74-7), which constitutes 'mathematical understanding'. In summarizing research in the area they claim the following benefits from 'learning and teaching with understanding':

Understanding is generative . . . Understanding promotes remembering . . . Understanding reduces the amount that must be remembered . . . Understanding enhances transfer . . . Understanding influences beliefs.

Although mathematics educators recognize that classroom construals and student perceptions of mathematics are significant issues, most research in the area lacks a clearly articulated sociological theory with which to link the structure of educational institutions, the power relations within these structures, the discursive ordering of knowledge in classroom mathematics, and student perceptions. Instead of looking to language and institutional culture for explanations, mathematics teachers are exhorted to foster a 'constructivist' approach which encourages students to make psychologically 'internal' connections between information (Cobb *et al.* 1992). In taking such a view, mathematics educators are denying themselves the insights offered by a sociological perspective.

Thus far, our linguistic exploration of the language of mathematics has revealed the way very distinctive kinds of technical meanings are

construed through mathematical language and has described the kinds of spoken language interaction which typify many mathematics classrooms. Of course, these two aspects of mathematics education are not unrelated. Many researchers have noted that there is a relationship between the kinds of linguistic interaction in which people engage and the kinds of meanings which can be realized by speakers. Such a relationship is fundamental to a range of socio-semiotic theories of meaning, including those of text and context in SFL (Halliday and Hasan 1985), semantic variation (Hasan 1996), speech genre (Bahktin 1986; Martin 1992) and semiotic mediation in higher mental functions (Vygotsky 1978, Volosinov 1983). A socio-semiotic approach requires us to examine closely the relationship between forms of expression and forms of knowledge. What is the relationship between the *kinds of interaction* between students and teachers that go on in the mathematics classroom and the *forms of consciousness* about mathematics that students develop as a result of their mathematics education?

In educational contexts this question begs to be reformulated as a sociological one: that of *differentiated access to meaning potential*. What role do language and classroom interaction play in providing some students with access to the technical meaning potential of mathematics while simultaneously denying access to others? Moreover, if we are considering intervention in mathematics education, we need to ask if the use of language currently found in mathematics classrooms is optimal for all students, and, if not, what the alternatives might be. It is in answering these questions that Bernstein's educational sociology is particularly useful.

Seen in Bernstein's terms, mathematics is a discipline whose discursive construction through language seems to be unusually closely aligned to the regulative discourse of the classroom and the macro-regulative discourse of the ordering of space and time in the school. Although Bernstein warns that 'it is very important to see that these discourses do not always move in a complementary relation to each other' (1996: 28), it appears that in school mathematics they do. There is a kind of synchronicity – a conspiracy if you like – between classification, instructional discourse, regulative discourse and language that is noticeably stronger than in other subject areas. This is evident both in what Bernstein calls the **classification** of knowledge in mathematics and the **framing** of pedagogic transmission of that knowledge.

Classification

Bernstein interprets the connectedness (or conversely, the separation) of knowledge as in terms of **classification**. For Bernstein (1996: 20–1, 24) classification is fundamentally about power:

Dominant power relations establish boundaries, that is relationships between boundaries, relationships between categories . . . the crucial space that creates

the specialization of the category is not internal to that discourse but is the space between that discourse and another. In other words A can only be A if it can effectively insulate itself from B . . . In the case of strong classification each category has its unique identity, its unique voice, its own specialised rules of internal relations . . . The arbitrary nature of these power relations is disguised, hidden by the principle of classification, for the principle of classification comes to have the force of the natural order and the identities that it constructs are taken as real, as authentic, as integral, as the source of integrity.

If there is a very strong classification between inside and outside then the knowledge here is given a special quality of otherness. If there is a strong classification between inside and outside then there is a hierarchy of knowledge classification between the so-called common sense and the so-called uncommonsense.

It is clear from both the differences between teacher and student language and in the way that mathematical language differs sharply from 'commonsense' forms of interaction that mathematics realizes a strongly classified discipline. Moreover, the calls from mathematics educators for greater interconnectedness of knowledge can be seen as calls for a weakening of classification. Since mathematics educators often claim that knowledge in 'real' mathematics (i.e. university research mathematics) is much more interconnected than in school mathematics, it would appear that the institution of schooling plays a major role in reshaping mathematical knowledge as strongly classified. Using Bernstein's insights we can posit that it is the need to make mathematical knowledge teachable and learnable within the power, space and time structures of the school and the need to make mathematical knowledge assessable through public examination, much more than individual teaching 'style', that makes school mathematics the way it is. Appeals to teachers to modify their 'traditional' teaching styles are, therefore, likely to meet with only modest success, since the teacher is not the only agent in the recontextualization process.

The implications of strong classification for mathematics education are many; however I shall illustrate them here with just one example. A classic issue of classification and recognition comes in the area of 'word problems' in mathematics. The status of word problems in relation to commonsense and uncommonsense knowledge is often somewhat ambiguous in mathematics syllabuses. In most teaching sequences word problems are added on to the end of the study of a topic. Teachers are told that word problems are challenging, help students to relate mathematics to the 'real world' and, above all, will be relevant to the way students will need to use mathematical knowledge outside of school. Yet word problems are nearly always contrived by teachers in order to fit within the parameters of the discipline. It is the uncommonsense, strongly classified discursive order in school mathematics which guides the selection and expression of word problems, not everyday, commonsense experience of the world. Knowl-

edge for outside is only permitted as long as it fits into the 'specialised rules of internal relations' of school mathematics. A highly predictable 'canon' of word problems thus emerges. The study of Pythagoras' theorem, for example, will always be accompanied by word problems about ladders being placed against walls or the use of compass bearings to work out how far someone has travelled; the study of perimeter will be accompanied by problems about fencing and marking the limits of a sports field; the study of area will be accompanied by problems about laying carpet or grass in a back garden, etc.

While most word problems can be effectively negotiated by most students while they remain within the strongly classified ordering of knowledge in the classroom, difficulties arise when word problems appear in random order, detached from an explicitly enunciated topic area in examinations. Many teachers with whom I have worked in Australia report that particular groups of students (generally people who are marginalized from the mainstream Anglo social milieu) have difficulty in recognizing the mathematical content of word problems which have been separated in space and time from their *uncommonsense* context – i.e. from the teaching of a particular topic in the classroom. These students are at particular risk in examinations and other assessment tasks, where space, time and language have removed the knowledge from the strongly classified context in which it was first introduced. This occurs for at least two reasons. First, many of these students simply do not recognize the so-called *commonsense* context construed by the problem – because mowing the lawn, travelling around the country, calculating income tax, etc. are not part of their everyday experience and secondly because sitting in an exam room reading word problems is *not* a commonsense context.

The misrecognition of the status of word problems is exacerbated by two official educational discourses, the first revolving around Piagetian notions of the concrete and abstract and the second around notions of the perceived advantages for less able students in doing 'real world' mathematics. It is frequently claimed that word problems will be easier because they represent concrete experience and that 'formal' mathematical knowledge is, by biological imperative, more difficult for students. By presenting situations which are assumed to be within students' everyday experience, the argument goes, students will be able to 'construct' 'internal' mathematical representations, rather than relying on 'external' 'imposed' forms of knowledge. Problems arise not from the viability or otherwise of this view, but from the mistaken assumption that the word problems that are produced in classroom mathematics actually constitute everyday forms of knowledge, and the denial of the role of language and symbols in semiotically mediating experience. It is assumed that the move from reading an 'everyday' problem to expressing an 'everyday' solution is one that does not require engagement with formal mathematical knowledge and will thus be easier for 'weaker' students.

The impact of economic rationalism on curriculum planners has also

resulted in a privileging of word problems in mathematics. The assumption here is that being able to do things with mathematics: calculate wages, overtime, taxes, interest rates, hire-purchase payments and exchange rates, and read graphs and tables, etc. is an economically more 'useful' kind of knowledge than 'pure mathematics' (the understanding of principles, axioms, formulae, etc).

The tendency of mathematics to be strongly classified has implications for the kinds of language-based intervention which one might attempt. As Martin (this volume Chapter 5) argues, reform is not simply a matter of weakening classification: this may simply act to render the discipline unteachable or to further disempower students by denying them access to highly valued forms of knowledge which are still being taught elsewhere. A more effective response would be to design teaching programmes which attempt to move backwards and forwards between strong and weak classification. Martin explains how some intervention programmes in Australia have attempted to do just this. Certainly, one very clear implication is to recognize that word problems, or any other kind of 'real world' maths that is introduced to the classroom is, by virtue of the fact that it is recontextualized as school learning, just as uncommonsense and esoteric as other more theoretical kinds of knowledge, and needs to be dealt with with the same explicitness as other kinds of knowledge. Simply leaving it to students to 'construct' 'internal' representations would seem an inadequate response, especially for marginalized students.

Framing

The patterns of spoken interaction described in the first part of this chapter make it clear that, as well as being strongly classified, the transmission, or framing, of mathematical knowledge is also tightly controlled in many classrooms. Framing, as Bernstein (1996: 27–8) sees it is all about control:

the form of control which regulates and legitimizes communication in pedagogic relations, the control of communication in local, interactional pedagogic relations [here teacher/pupil and pupil/pupil]. Framing refers to the nature of the control over:

- the selection of the communication;
- its sequencing (what comes first, what comes second)
- its pacing (the rate of expected acquisition)
- the criteria, and
- the control over the social base which makes this transmission possible

... We can distinguish analytically two systems of rules regulating the framing. And these rules can vary independently of each other ... These are rules of social order and rules of discursive order. We shall call the rules of the social order regulative discourse and the rules of discursive order instructional discourse. And we shall then write this as follows:

framing = instructional discourse
regulative discourse

In other words, the instructional discourse is always embedded in the regulative discourse, and the regulative discourse is the dominant discourse.

SELECTION AND SEQUENCING

In examining mathematics syllabuses it becomes clear that the strongly classified nature of the knowledge necessitates a high degree of control over the selection and sequencing of content. Only with careful control of selection and sequencing can the knowledge be rendered teachable. Of course, this degree of control serves only to further strengthen the classification of mathematical knowledge – strong classification and strong framing thus feed off one another. Examples of carefully controlled selection and sequencing can be found in just about all areas of mathematics syllabuses. In all junior secondary school syllabuses in Australia, for instance, basic co-ordinate geometry is taught. Students are taught about ordered pairs (x, y) and how to plot these pairs on the number plane. At this point the uncommonsense content is often cloaked in the guise of relevance and fun, with activities such as map reading, locating places with latitude and longitude, finding grid references on a street directory and looking for hidden treasure on an island. A bit later on, map reading and treasure hunts give way to plotting basic functions on the number plane (straight lines, circles and possibly even parabolas). What is the purpose of this selection and this sequence? Not to make students better map-readers or treasure hunters, but to provide students with the requisite skills for the study of polynomials and calculus in the senior years. The rules of the discursive order within a strongly classified discipline thus shape selection and sequencing of learning experiences offered up to students, no matter how everyday or relevant they are dressed up to be.

PACING AND CONTROL OVER THE SOCIAL BASE

In the area of pacing and control over the social base it is clear that there is a close relationship in mathematics between the social order of the school, the regulative discourse, and forms of the mathematical knowledge itself. This is shown most clearly in catechistic teacher–student interactions. Consider the following transcription from a Year 7 classroom:

Text 5

- T: Okay, class. Books out. There's five questions going on the board. You have one minute.
P: Okay, Sir.
(Sounds of chalk on board)
P: How come you've got a . . .

- T: Would you like it?
Ps. Yeah.
T: We might do a little bit more of that later on.
P: Oh, great.
P: When? Today?
T: No, not today.
P: Sir, what about our test results?
T: Test results. There's a couple of people still to do them in other classes so I can't give them back just yet.
(Class noise)
T: Come on, mate.
P: Sir, when are we going to get our test results back?
(Class noise)
T: No I want these answers in the books.
P: About the sheets, Sir?
T: No, I'll tell you all about the sheets later.
. . .
T: Troy. Are you finished?
P: Yeah.
T: Okay, Troy, the first one.
P: (undecipherable) 0.8.
T: 0.8. Correct. Second one, Craig.
P: I haven't even started.
T: Quick, do it in your head. 6.6 divided by 3.
P: 2.2.
T: Why is it 2.2?
P: (undecipherable)
T: Don't look at other people's answers. Helen?
P: 2.2.
T: 2.2. Yes. Craig. 6 divided by 3 is 2. 6 divided by 3 is 2. 2.2. Third answer . . . Girl. Kelly?
P: I don't have that one yet, but I done the last one.
T: Do it in your head then, number 3.

Like the interaction examined earlier in Text 3, the teacher in this transcript consistently occupies the role of primary knower, posing and then evaluating questions. The difference here is that much of the IRE interaction is not about knowledge, but about the control of the *physical activity* of students in the classroom:

- T: Troy. Are you finished?
P: Yeah.
T: Okay.

This control of *physical activity* is seamlessly integrated with control over the knowledge:

- T: Second one, Craig.
P: I haven't even started.

- T: Quick do it in your head. 6.6 divided by 3 .
 P: 2.2.
 T: Why is it 2.2?
 P: (undecipherable)
 T: Don't look at other people's answers. Helen?
 P: 2.2.
 T: 2.2. Yes.

Thus the regulation of physical behaviour and the construal of mathematical knowledge are virtually indistinguishable from one another in the pedagogic discourse. In such a context, it is not really surprising that many students may see mathematics as 'a necessary set of rules and procedures to be learned by rote'. In the light of Text 4 it can be seen that the IRE exchanges in the Year 11 class, illustrated in Text 2, while being overtly about the exchange of information, are also, by virtue of their highly ritualized form, about the control of physical behaviour. The basis for the forms of knowledge that are construed in mathematics would appear to lie just as much in the social order of classroom life as they do in the discursive order of the discipline.

When it comes to the question of appropriate intervention, there are once again no clear cut, simplistic solutions. An obvious response to the strong framing evidenced by Texts 2 and 4 would be to seek communicative alternatives to the IRE style of teacher-student exchange. This seems to be what is motivating the strong push towards group-based work in mathematics education. Cobb *et al.* (1991: 7) advocate the following role for the teacher:

The teacher's role in initiating and guiding mathematical negotiations is a highly complex activity that includes highlighting conflicts between alternative interpretations or solutions, helping students develop productive small-group collaborative relationships, facilitating mathematical dialogue between students, implicitly legitimizing selected aspects of contributions to discussion in light of their potential fruitfulness for further mathematical constructions, redescribing students' explanations in more sophisticated terms that are nonetheless comprehensible to students, and guiding the development of taken-to-be-shared interpretations when particular representational systems are established.

While Cobb *et al.* offer a well-meaning alternative to the oppressive regime of the strongly classified, strongly framed pedagogy of many mathematics classrooms, their vision of the mathematics classroom is largely asociological and predicated on a number of assumptions about students' linguistic and social habits. Where for example do 'alternative interpretations or solutions' come from if students do not have the linguistic skills to generate them? An analysis of student-student interaction, such as Text 4, suggests that even talented senior students in socio-economically disadvantaged schools cannot readily use language to present clearly articulated

alternative interpretations or solutions, even though they are prepared to debate isolated *answers* among one another. What counts as 'productive small-group collaborative relationships' and which social groups are more likely to fall easily into this kind of compliant behaviour? Will all students be equally able to read the hidden message in the 'implicitly legitimizing [of] selected aspects of contributions to discussion in light of their potential fruitfulness for further mathematical constructions'? Which students will pick up on the teacher's 'more sophisticated terms' when she 'redescribes students' explanations'? The continued poor performance of socio-economically disadvantaged students in classrooms where the response has been simply to weaken framing suggests that not all students stand to gain equally from a simplistic interpretation of the constructivist vision.

Perhaps a more responsible approach (and probably one which would be approved of by constructivists) would be the careful scaffolding of knowledge through a process of guided interaction. The implications of this in terms of framing are discussed by Martin (this volume Chapter 5). Christie (1998) has put forward a clear case for different kinds of interaction, including IRE exchanges, at different points in the teaching/learning process.

Conclusion: the role of a sociologically informed educational linguistics in mathematics education

My purpose in writing this chapter has not been to offer easy solutions to issues in mathematics education, nor has it been to offer details of language-based intervention in mathematics classrooms.⁶ Rather I have sought to make clear what it is that a socio-semiotic model of language, such as SFL, and an elaborate theory of the sociology of education, such as Bernstein's, can offer mathematics educators. Neither socio-linguistics nor sociology appear to have made a particularly strong impact on mathematics education in the past, both because of the strongly classified nature of mathematics education (like the discipline itself) and the privileging of psychological models of learning in mathematics, such as that of constructivism. It is important to note that socio-linguistic and sociological contributions do not necessarily entail a rejection of psychological models, although they may challenge some of the assumptions of psychological models, or at least bring to the foreground issues which do not receive great emphasis in psychological models.

As I see it, socio-linguistic and sociological models can make the following contributions:

1. Make explicit the kinds of language which are employed to construe the technical meanings of mathematics. This description goes well beyond the identification of technical lexis and phrases and includes grammatical structures, discourse analysis and the identification of spoken and written genres.

2. Allow us to understand clearly differences in linguistic behaviour among different language users and in different social contexts within the mathematics classroom.
3. Provide clear direction for language-based intervention in mathematics; a clear but not atomistic focus for language-based activities; and clear criteria for effective language use which can be shared by teachers and students.
4. Allow for a detailed analysis and critique of current linguistic practices in mathematics education, especially in teacher language, interaction patterns, written resources and public examinations.
5. Provide a way of linking broader sociological factors, such as power, space and time relationships in the school, to the way in which mathematical knowledge is construed in the classroom and perceived by students.
6. Expand the critical tools available to mathematics educators to analyse the kinds of intervention they seek to encourage in the classroom. In particular those that can effectively complement psychological models.

In helping us to understand these areas of concern, socio-linguistic and sociological models constitute a worthwhile tool for investigation and intervention in mathematics education.

Notes

- 1 'Uncommonsense', following Bernstein (1977, 1990), is being used here to distinguish educational knowledge from 'commonsense', or 'everyday' knowledge. The latter is accumulated through our experience of the physical world on a day-to-day basis, whereas the former is usually acquired through formal education, which reformulates knowledge from its 'primary context' (mainly through language) according to the principles and relations of the pedagogic context.
- 2 Although some researchers have decided that IRE exchanges are undesirable because they are not 'genuine' and have exhorted teachers to avoid them, others, notably Christie (1998), have argued for their functional status in certain stages of classroom discourse.
- 3 One mathematics teacher told me the story of a student who, having being presented with an equation and told to 'find x ', drew an arrow pointing to the symbol x and wrote 'here it is'!
- 4 Christie, Martin and Rose all discuss grammatical metaphor in this volume.
- 5 Halliday identifies 'interlocking definitions' as an aspect of *scientific* language, but curiously he uses mathematical terms (radius, centre, circle, circumference, diameter) as an example (Halliday 1993b).
- 6 Though this has certainly been done using SFL (Veel 1994).

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