

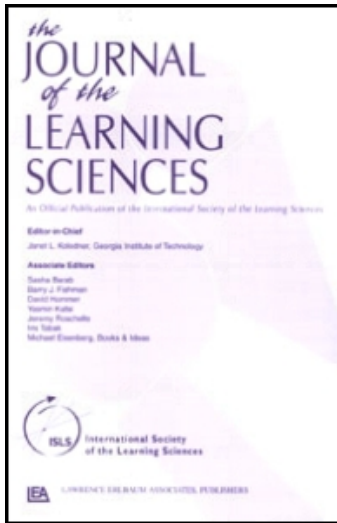
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Flowing Toward Correct Contributions During Group Problem Solving: A Statistical Discourse Analysis

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Flowing Toward Correct Contributions During Group Problem Solving: A Statistical Discourse Analysis

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Groups that created more correct ideas (correct contributions or CCs) might be more likely to solve a problem, and students' recent actions (micro-time context) might aid CC creation. 80 high school students worked in groups of 4 on an algebra problem. Groups with higher mathematics grades or more CCs were more likely to solve the problem. Dynamic multilevel analysis statistically identified watersheds (break-points) that divided each group's conversation into distinct time periods with many CCs versus few CCs, and modeled the groups' 2,951 conversation turns. Wrong contributions, correct evaluations of one another's ideas, justifications, and polite disagreements increased the likelihood of a CC. In contrast, questions, rude disagreements, and agreements reduced it. Justifications had the largest effects, whereas the effects of correct evaluations lasted 3 speaker turns. Some effects differed across groups or time periods. In groups that solved the problem, justifications were more likely to yield CCs, and questions were more likely to elicit explanations. Meanwhile, the effects of agreements and correct evaluations on CCs differed across time periods. Applied to practice, teachers can encourage students to evaluate others' ideas carefully and politely, express and justify their own ideas, and explain their answers to group members' questions.

GROUP PROBLEM-SOLVING PROCESSES

Past research suggests that groups with more CCs are more likely to solve a problem correctly than other groups (*functional theory of group decision-making*; e.g.,

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Orlitzky & Hirokawa, 2001). Let us call this Hypothesis 1 (H-1). (Hypotheses are numbered according to their level of analysis: group, 1; time period, 2; speaker turn, 3.)

H-1. Groups with more CCs are more likely to solve the problem correctly.

Successful group problem solving might yield more clusters of CCs via diverse ideas and argumentation (Amason, 1996; Cobb, 1995). However, rude arguments might hinder CCs and group problem solving, especially arguments centered on status struggles. This study investigated CCs by examining whether they clustered together and by identifying group problem-solving processes that helped or hindered the creation of CCs.

Clusters of CCs in Time Periods

Several researchers have claimed that group problem solving tends to be more successful if a group steps through each formal problem-solving phase in order (*phase model*), such as by (a) clarifying the problem, (b) discussing criteria, (c) proposing solutions, and (d) evaluating proposals (Ellis & Fisher, 1994; Pavitt, 1993). In such a model, components of a correct solution are split across the different phases. Group members likely voice correct and incorrect ideas at each stage (barring an algorithmic march to a correct solution). Hence, successful groups with multiple phases might tend to have clusters with many CCs alternating with clusters with few CCs.

However, groups often do not step through each phase (Hirokawa, 1983; Pavitt & Johnson, 2001). Instead, many groups prefer to discuss one solution proposal in full, then another, and so on (*reach-testers*; Pavitt & Johnson, 2001). These reach-tester groups might have all CCs clustered around the correct proposal and incorrect ideas elsewhere, yielding only one or two distinct time periods. As phase model groups are more likely to be successful than reach-tester groups, CCs might occur in more clusters in successful groups rather than in unsuccessful ones, especially for difficult problems.

H-2. CCs occur in many clusters in successful groups, but in fewer clusters in unsuccessful groups.

Group Problem-Solving Actions That Help Create CCs

Compared to individuals, group members' diverse perspectives and argumentation might create more CCs (Cobb, 1995; Paulus & Brown, 2003). Diverse points of view can help a group create more ideas and judge them more accurately compared to individuals (Paulus & Brown, 2003).

New ideas. Group members often have diverse perspectives and sources of knowledge (Stasser, 1992). Capitalizing on this diversity, heterogeneous groups often create many ideas, representations, and solution proposals, thereby raising the likelihood that at least one of their ideas is correct/optimal (Paulus & Brown, 2003).

Group members can express idiosyncratic ideas and build on them to create new alternatives through processes such as sparked ideas, jigsaw pieces, and creative misinterpretations (Chiu, 1997; Paulus & Brown, 2003). Comments by one person (e.g., a key word) might spark another person to activate related concepts in his or her semantic network and propose a CC (Nijstad, Diehl, & Stroebe, 2003). Or, two or more members can put together different pieces to construct a CC, like fitting jigsaw pieces together (Chiu, 1997). Finally, a person might misinterpret a group member's incorrect idea to create a new, correct one (Chiu, 1997). Thus, even wrong contributions can lead to CCs.

Group members' diverse views also help them recognize flaws, refine these incorrect ideas, and create CCs (Cobb, 1995; Piaget, 1985). Groups with diverse views might create more wrong ideas, but their diverse views also improve their judgment of their validity. Hence, they can detect and correct these flaws to create CCs. This contrasts with the view that people primarily build on correct ideas and that wrong ideas often lead the group astray (see "Group Problem-Solving Processes" column [hereafter, *middle column*] of Figure 1, which summarizes the hypotheses and their relationships).

H-3a. Contributions, including wrong contributions, help create CCs.

Argumentation. Successful group problem solving often involves argumentation in the cognitive/problem content space (Roschelle, 1992), a social process by which people explain and justify their own views to convince both themselves and others (Amason, 1996; Cobb, 1995). During this process, group members evaluate one another's ideas, detect flaws, and justify their ideas (Cobb, 1995; Kuhn, Shaw, & Felton, 1997). These argumentation processes can help students develop their understanding of the specific content (e.g., algebra's structural relationships among equality, arithmetic operations, and properties of number; Kieran, 1992).

Evaluations characterize how a person assesses the previous speaker's action and problem-solving approach (*functional theory of group decision-making*; e.g., Chiu, 2000a, 2001; Orlitzky & Hirokawa, 2001). For example, Sean says, "Three times four is seven." Maya can agree ("Uh-huh"), use a neutral action ("What did you say?"), disagree ("Nope, you're wrong"), or change the topic ("When is class over?"). Whereas agreements continue the current problem-solving trajectory, disagreements and changes of topic try to change the trajectory (Chiu, 2001). Hence, these evaluations reflect the accountability of a person's ideas and actions to his or her learning community, to its collective knowledge, and to its local standards of reasoning (e.g., validity of transformations among equivalent algebraic expressions; Michaels, O'Connor, & Resnick, in press).

Evaluations can also be right or wrong in many contexts (e.g., high school algebra). Correct evaluations support correct ideas (“Three times four is twelve, right”) or identify flawed ideas (“No, three times four is not seven”), thereby creating a foundation of partially shared understandings of correct ideas that group members can use to build new CCs. In contrast, incorrect evaluations reject CCs (“Nope, five times two isn’t ten”) or accept flawed ideas (“Three times four is seven, yeah”), embedding flaws in their partially shared understandings. Group members using these partially shared understandings can carry these flaws into their new ideas, resulting in more wrong contributions and fewer CCs. A group’s collective attention and diverse perspectives can help it evaluate ideas correctly and create a partially shared foundation of understanding to aid creation of CCs (see Figure 1, middle column; Cobb, 1995; Hinsz, 1990).

According to *sociocognitive conflict theory*, group members can recognize problems or difficulties (perturbations), express them through disagreements or questions, and address them to improve their understanding (Doise, Mugny, & Perret-Clermont, 1975; Piaget, 1985). Piaget defined two types of perturbations: (a) obstacles, which give negative feedback; and (b) lacunae, or gaps in understanding. Thus, disagreements can identify obstacles (e.g., “No, that’s wrong, three times four isn’t seven”) and motivate the need to create CCs.

Meanwhile, a question (e.g., Juan asks, “How do we find the speed?”) can indicate an individual gap or a group gap. If the gap is an individual one, other group members who know the answer can explain it. Thus, individual gap questions invite explanations that often review previous ideas rather than create new CCs. In contrast, no one knows the answer to a group gap question, which motivates the need for a CC and points to a new direction for creating it. By expressing their ideas and explanations, students open up their reasoning for group members to analyze and discuss (Franke, Carpenter, & Battey, 2007). De Lisi and Goldbeck (1999) argued that group members’ diverse perspectives and levels of knowledge facilitate both perturbations and responses to them. In short, these perturbations can motivate and inform the creation of more CCs (see Figure 1, middle column).

Both disagreements and questions invite justifications that establish an idea’s validity within the local classroom community’s negotiated norms for a specific content area (e.g., algebraic relationships and mathematics proofs; Balacheff, 1988). Chiu and Khoo (2003) showed that members of successful groups often anticipated criticisms and justified their new ideas. Likewise, after a person disagrees with a proposal (e.g., Maya), the original proposer (Sean) might justify the proposal by linking it to data, using a warrant, or supporting a warrant with backing (Toulmin, 2003). Along with appeals to external authorities (e.g., teacher, textbook), mathematics also allows students to build on example-based justifications by generalizing them to create principled, internal justifications of structural relationships within a closed system (Sowder & Harel, 1998). In response, other members can give different views and justifications (Piaget’s, 1985, genuine argument).

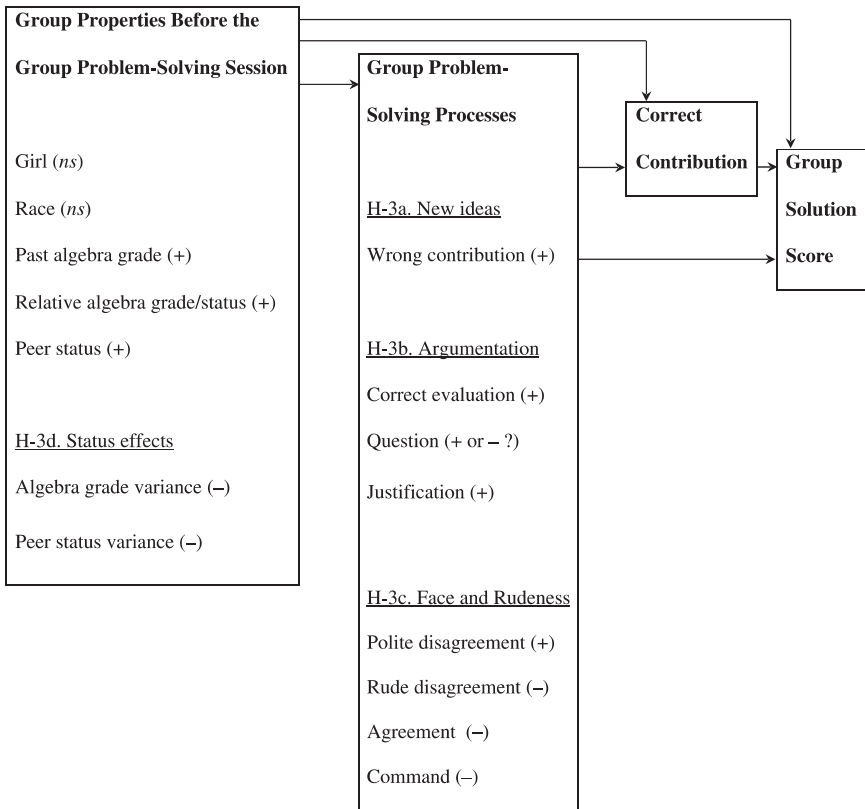


FIGURE 1 Model of the effects of student and group properties before and during the group problem-solving process on the outcome variable correct contributions and on group solution score. Symbols in parentheses indicate the expected direction of the relationship with the outcome variables: positive [+], negative [-], or not significant [*ns*]. H = hypothesis. F

Similarly, when Juan shows a gap in understanding by asking a question, other members can respond with explanations and justifications (Coleman, 1998). As justifications support an idea's validity, they can help create CCs (see Figure 1, middle column; e.g., Goldbeck, 1998).

H-3b. Correct evaluations, group knowledge gap questions, and justifications facilitate the creation of CCs.

Group Problem-Solving Actions That Hinder CCs

Disagreements can help create CCs according to sociocognitive conflict theory, but the effects might differ for polite and rude disagreements according to *polite-*

ness theory (Brown & Levinson, 1987; Chiu, 2001). Polite disagreements likely facilitate CCs and group problem solving, but rude disagreements can hinder them, especially during status struggles. When arguments spill over from the problem content space into the social relational space (Barron, 2003), group members might sacrifice further problem-solving progress in favor of protecting their public self-images (*face*; Brown & Levinson, 1987; Chiu & Khoo, 2003). Status differences can further aggravate these face concerns.

Face and rudeness. As problem solving occurs in the dual space of problem content and social relations, each type of evaluation can affect both the problem solving (as noted above) and the previous speaker's face (Chiu, 2000b, 2001). Evaluations range from polite to rude: agreement, neutral, change of topic, and disagreement (Brown & Levinson, 1987; Chiu, 2000b, 2001). Consider Sean's utterance again: "Three times four is seven." If Maya agrees with Sean ("Uh-huh"), she supports him, promotes his face, and enhances their social relationship (Brown & Levinson, 1987). Thus, members often repeat shared information to create common ground and solidarity (Clark & Brennan, 1991). Moreover, people spontaneously reciprocate positive affective displays, such as eye contact, to suggest agreement with one another (Burgoon, Dillman, & Stern, 1993).

In contrast, other actions do not support face. Neutral actions include discourse management or meta-discourse actions (e.g., "What did you say?"). Although changes of topic ("When is class over?") can be neutral, they can be rude if the previous speaker (Sean) expects a response (e.g., if Sean asks the question "Three times four is seven?"). If Maya says "When is class over?" after Sean's question, she either ignores him or does not listen to him, both of which are rude. Lastly, disagreements (e.g., "No, you're wrong") can threaten face by lowering public perception of the previous speaker's (Sean's) competence (Brown & Levinson, 1987).

When a person disagrees (e.g., Maya says, "Nope, you're wrong"), the target person (Sean) ideally tries to understand the criticism and use the information to create a CC. However, the threat to Sean's face may encourage him to retaliate emotionally (*face attack*; "No, I'm not! You are. You're always making mistakes in math...." Chiu & Khoo, 2003; Tracy & Tracy, 1998). Thus, rude disagreements threaten face, escalate interpersonal conflict, and often hinder the creation and recognition of CCs (see Figure 1, middle column). In the worst case, a spiral of rude disagreements can kill the collaboration. Even if the collaboration survives after a rude disagreement (or some other rude action; e.g., insult), group members might withhold CCs or correct evaluations rather than risk losing face (Chiu & Khoo, 2003).

To avoid threatening Sean's face, Maya might go to the opposite extreme and publicly agree. By doing so, Maya enhances her social relationship with Sean at the expense of their problem solving. Such false agreements allow errors to persist and potential CCs to remain unspoken (see Figure 1, middle column). For exam-

ple, teenage girls often avoid disagreeing with one another (Tudge, 1989). Even authority does not eliminate this effect, as tutors often do not point out their students' errors (Person, Kreuz, Zwaan, & Graesser, 1995).

Avoiding the extremes of rude disagreement and false agreement, Maya can disagree politely (with redress) to reduce the threat to Sean's face and maintain problem-solving integrity (Brown & Levinson, 1987; Chiu & Khoo, 2003). Instead of "No, you're wrong," Maya can disagree politely, "If three is multiplied by four, we don't get seven." The polite disagreement both reduces blame and creates common ground. First, Maya uses the hypothetical "if," thereby distancing the error away from them. Second, she does not refer to Sean, (no "you"), thereby avoiding assignment of blame. Third, Maya uses the passive voice ("is multiplied") not the active voice, to hide causal agency and responsibility. Lastly, she uses the passive circumstantial verb "get," thereby implicating agency in external conditions.

Maya's polite disagreement creates common ground by repetition and shared positioning. By repeating Sean's computation "Three is multiplied by four . . . seven," Maya suggests that she shares his understanding. Maya also uses shared positioning ("we") to claim common cause with Sean.

Maya's polite disagreement supports her relationship with Sean, so he is less likely to retaliate. Instead, Sean is more likely to try to understand Maya's criticism, recognize the flaw, and correct it with a CC (Chiu & Khoo, 2003). Indeed, the benefits of polite disagreements are so strong that it is the accepted norm among peers, as lack of redress during a disagreement is noticeably rude and unacceptable (Holtgraves, 1997). In short, polite disagreements might support social relationships, CC construction, and CC recognition, thereby enhancing both the problem content and social relational spaces (see Figure 1, middle column).

Other rude actions include commands and insults. As commands demand action from the target listener(s), they impinge on the target listener's freedom and are less polite than questions or statements (see Figure 1, middle column). Likewise, insults attack the target listener's face (Tracy & Tracy, 1998).

H-3c. Polite disagreements help creation CCs, but rude disagreements, false agreements, and commands hinder CC creation.

Status. According to *status characteristics theory*, status differences can reduce CCs and distort evaluations through the pursuit of high status via status struggles (Bales, 2001; Gersick, 1988) or through the greater influence of high-status members (Cohen, 1994). Cohen (1994) defined *status* as "an agreed-on rank order where it is generally felt to be better to be high than low rank" (p. 23).

As a higher status person often receives more group resources and attention, people often compete for higher status (status struggles), especially if no clear status hierarchy exists (Bales, 2001; Gersick, 1988). During status struggles, intentional rude disagreements can hinder creation of CCs, but they can also enhance

one's face by forcing a competitor to lose face (face attacks; e.g., "Five times two is *obviously* ten, not seven"; Tracy & Tracy, 1998).

After a status hierarchy has been established, group members expect higher status members to have greater task competencies and to contribute more toward the desired outcome(s) (Dembo & McAuliffe, 1987). As a result, higher status members have more opportunities to perform and receive rewards, as others selectively invite and defer to their opinions while discouraging, undervaluing, or outright ignoring lower status members' ideas. Thus, excessive attention to status can distort evaluations toward excessive agreement with higher status members. By doing so, group members enact their expectations of high-status members dominating the interaction and might increase the ratio of flaws to correct ideas in their partially shared understandings.

High-status members' influence can also increase over time. High-status people tend to speak early and often (Hackman & Johnson, 2000). As group members value and prefer supporting previously discussed, shared information rather than introducing new, unshared information (Stasser & Titus, 1985), high-status members' domination increases in severity over time (Stasser & Taylor, 1991).

Greater status differences might increase the incentives for status struggles and yield greater status effects, both of which might reduce CCs (see Figure 1, left column). For group problem solving among students, the primary status characteristic is often past achievement, but group members might also use diffuse status characteristics (e.g., race, gender) to make assumptions about one another's competence (Cohen, 1982; Webb, 1984).

H-3d. Greater differences among group members' statuses (achievement, peer status, gender, or race) reduce CCs.

Successful Versus Unsuccessful Groups

Considering these group processes, groups with four properties might be more likely than groups without these properties to solve a problem successfully. First, groups with more CCs might be more likely to succeed than other groups. Second, phase model groups might be more likely to have their CCs occur in clusters and to succeed compared to reach-tester groups. Third, groups that more often engage in processes that help create CCs might be more likely to succeed. Fourth, groups that show more rude behaviors or have larger status differences (processes that hinder CCs) might be less likely to succeed.

In sum, this study examined the process of CC creation during group problem solving by testing the following hypotheses. First, groups with more CCs are more likely to solve the problem correctly. Second, CCs occur in more clusters in successful groups than in unsuccessful ones. Third, correct and wrong contributions help create CCs. Fourth, correct evaluations, group knowledge gap questions, and

justifications aid CC creation. Fifth, polite disagreements help create CCs, but rude disagreements, false agreements, and commands hinder CC creation. Lastly, greater status differences hinder CC creation.

METHOD

Using videotapes and transcripts described in Chiu and Khoo (2003, 2005, in press), this study addressed a different research question with a different outcome variable (CCs). In Chiu and Khoo's (2003) study, the variation in student evaluations of one another's ideas during group problem solving occurred mostly at the speaker turn level rather than at higher levels (e.g., group or classroom). Thus, this study focused on simpler, more proximal analyses of speaker turns, time periods, and groups (leaving more complex, distal analyses involving classroom and school differences for future studies).

I analyzed the data at the group, time period, and speaker turn levels to model problem-solving outcomes and processes. Representative transcript segments illustrate the relationships among variables.

Participants

The participants attended four ninth-grade algebra classes in an urban U.S. high school that scored at the 40th percentile in mathematics (maximum = 100; California Department of Education, 2005). In all, 87 students were asked to answer a peer status survey and to be videotaped. Of the 87 students, 7 (or 8%) declined to participate. (Of these 7 students, 4 were girls and 3 were boys. Their average grade was 77/100.) There were 40 girls and 40 boys. By race, there were 12 Asians, 27 Blacks, 28 Hispanics, and 13 Whites.

These students worked in groups of four. There were no same-gender groups and no same-race groups. These students had attended the same algebra class for 7 months and were likely aware of one another's mathematics abilities through conversations inside and/or outside of class. However, these students had not received any group work training and had not previously worked together in groups. Thus, group members' relative mathematics abilities were more likely to have a primary status effect. Likewise, diffuse status characteristics such as gender and race were likely to have smaller effects compared to those of strangers (Sharan & Shachar, 1988; Webb, 1991).

Procedure

All 80 algebra students who agreed to participate answered two questions regarding peer status: "Who are 3 classmates you would most like to hang out with?" and

“Name 3 classmates who are the easiest for you to talk with outside of school work.” Later, their teacher presented the following problem in their algebra classes:

You won a cruise from New York to London, but you arrive 5 hours late. So, the ship left without you. To catch the ship, you rent a helicopter. The ship travels at 22 miles an hour. The helicopter moves at 90 miles an hour. How long will it take you to catch the ship?

As advocated by cooperative learning researchers (e.g., Cohen, 1994; Johnson & Johnson, 1994), this problem was challenging for these groups of students and had multiple solution methods (see Appendix A). The classes had studied equations with single variables, and the teacher used the above problem to introduce them to a new unit on algebraic equations with multiple variables. Hence, the students had not yet learned, in class, any procedures for solving this problem. Furthermore, the problem involved complicated mathematics relationships, nontrivial combinations of algebraic operations, and a noninteger solution. One solution equates the distance computations for each vehicle (cruise ship and helicopter; $22 \text{ mph} \times [\text{Time} + 5 \text{ hr}] = 90 \text{ mph} \times \text{Time}$) to obtain 1.618 hours or 1 hour 37 minutes.

The students worked in groups for 30 minutes. (If students finished early and chatted off task, this off-task talk was not included in the analysis. Only groups that successfully solved the problem finished early.) They had pens, paper, and calculators available for their use. There were six to seven video cameras in a classroom, one following the teacher and one for each group of students. Likewise, the teacher and each group of students had their own microphone and audiotape recorder to back up the video recordings. The videotaped data were transcribed, coded, and analyzed.

Variables

See Table 1 for summary statistics and descriptions of variables. Using a similar set of data from a pilot study, I trained two research assistants to transcribe and code the videotapes. Each transcript was divided into sequences of words or actions (e.g., writing “3 + 40”) by the same person (*speaker turns*). Blind to the study’s hypotheses, the research assistants coded each speaker turn from the videotape onto a transcript, maintaining a log of each videotape to aid their coding. To compute the interrater reliability, I used Krippendorff’s (2004) α . Unlike other reliability measures, Krippendorff’s α applies to any number of coders, any number of categories or scale values, any level of measurement, any sample size, and incomplete data. Its values range from -1 (maximum disagreement) to 1 (perfect agreement). A value near 0 indicates chance agreement, and a value of $.7$ or higher indicates satisfactory agreement.

TABLE 1
Summary Table of Group-Level Variables

| <i>Variable</i> | <i>M</i> | <i>SD</i> | <i>Min</i> | <i>Max</i> | <i>Description</i> |
|-----------------------------------|----------|-----------|------------|------------|---|
| Outcome variables | | | | | |
| Solution score | 1.90 | 1.25 | 1 | 3 | Score of group's final solution (see Appendix A) |
| Correct contribution | 0.28 | 0.18 | 0.02 | 0.59 | A correct idea that has not been mentioned earlier during the group problem-solving session |
| Before problem solving | | | | | |
| Classroom_1 | 0.25 | 0.44 | 0 | 1 | Binary variable for students in Classroom 1. Baseline classroom is Classroom 4. |
| Classroom_2 | 0.30 | 0.46 | 0 | 1 | Binary variable for students in Classroom 2 |
| Classroom_3 | 0.25 | 0.44 | 0 | 1 | Binary variable for students in Classroom 3 |
| Girl | 2.00 | 0.65 | 1 | 3 | Number of girls in each group (0 indicates all boys) |
| Asian | 0.60 | 0.50 | 0 | 2 | Number of Asians in each group |
| Latino | 1.40 | 0.75 | 0 | 4 | Number of Latinos in each group |
| White | 0.65 | 0.49 | 0 | 2 | Number of Whites in each group |
| Mathematics grade | 82 | 7 | 71 | 92 | Mean of last semester's mathematics grades for all students within a group |
| Highest mathematics grade | 92 | 8 | 77 | 99 | Highest mathematics grade of any student within a group |
| Peer status | 23 | 8 | 9 | 37 | Mean number of times a student's name appeared in classmates' answers to the following questions: "Who are 3 classmates you would most like to hang out with?" and "Name 3 classmates who are the easiest for you to talk with outside of school work." This measure is the mean for the group and serves as a proxy for the group's social skills. |
| Measures of status effects | | | | | |
| Math grade variance | 101 | 70 | 12 | 300 | Variance of students' mathematics grade within each group |
| Peer status variance | 37 | 29 | 0.25 | 108 | Variance of peer status within each group |

| | | | | | | | | | |
|------------------------|-------|-------|------|-------|---|--|--|--|--|
| During problem solving | | | | | | | | | |
| Total no. of words | 1,363 | 1,174 | 371 | 3,885 | Total number of words during a group's problem-solving sessions. The large standard deviation is partially due to five groups that spoke more than 2,400 words each. | | | | |
| Total on-task words | 1,338 | 1,277 | 342 | 2,841 | Total number of words spoken on on-task turns during a group's problem-solving sessions (see description for Total no. of words) | | | | |
| New ideas | | | | | | | | | |
| Wrong contribution | 0.12 | 0.05 | 0.04 | 0.20 | A flawed idea that has not been mentioned earlier during the group problem-solving session | | | | |
| Argumentation | | | | | | | | | |
| Correct evaluation | 0.37 | 0.19 | 0.14 | 0.79 | Agreement with the previous speaker's correct idea or disagreement with the previous speaker's incorrect idea | | | | |
| Unresponsive | 0.16 | 0.09 | 0.02 | 0.31 | Ignoring of the previous speaker; initiation of new topic | | | | |
| Polite disagreement | 0.16 | 0.06 | 0.06 | 0.27 | Disagreement with the previous speaker with at least one form of redress | | | | |
| Question | 0.23 | 0.07 | 0.15 | 0.45 | An elicitation that expects a verbal response or a nonverbal substitute (Tsui, 1992; Sinclair & Coulthard, 1992) | | | | |
| Justification | 0.12 | 0.09 | 0.04 | 0.30 | An action that supports an answer or claim by at least one of the following: linking it to <i>data</i> , using a <i>warrant</i> , or supporting by <i>backing</i> (Toulmin, 2003) | | | | |
| Face and rudeness | | | | | | | | | |
| Rude disagreement | 0.09 | 0.05 | 0.02 | 0.19 | Disagreement with the previous speaker without redress | | | | |
| Agreement | 0.58 | 0.10 | 0.39 | 0.86 | Agreement with the previous speaker | | | | |
| Command | 0.06 | 0.07 | 0.00 | 0.20 | A directive that invites a nonverbal response (Sinclair & Coulthard, 1992) | | | | |

The research assistants tried to settle coding disagreements by consensus. They could not agree in 19 cases, so I made the final coding decision. Due to poor sound quality, 49 speaker turns could not be coded. These turns were coded as *missing* and inspected with adjacent outcomes and predictors for significant correlations. As they did not correlate significantly with other variables, omitting them likely did not affect the results.

Speaker turn variables. Unlike flat classification schemes that only allow one or two codes for each speaker utterance (e.g., Bales, 2001), the research assistants coded each speaker turn along five dimensions: evaluation of the previous action, knowledge content, validity, justification, and invitational form (Chiu, 2000a; Chiu & Khoo, 2003). Evaluation of the previous action, knowledge content, and invitational form captured interactions and relationships across speaker turns (*relational* measures). (See Tables 1 and 2 and Appendix B for coding examples, coding decision trees, and further details.) As the data had only two insults, the statistical power was too low to test their effects.

Data Analysis

A group-level analysis tested the relationship between CCs and a correct solution. I describe this analysis and then discuss some difficulties with lower level analyses and strategies for addressing them. Afterward, I identify watersheds (breakpoints) and time periods, followed by speaker turn level analyses of CCs. (See Table 3 for a summary of the hypotheses, data, models of variables, and theoretical rationales.)

TABLE 2
Coding of a Classroom Discourse Segment

| <i>Person</i> | <i>Action</i> | <i>EPA</i> ^a | <i>KC</i> ^b | <i>Validity</i> ^c | <i>Justification</i> ^d | <i>IF</i> ^e |
|---------------|--|-------------------------|------------------------|------------------------------|-----------------------------------|------------------------|
| Ana | Do three times four hours. | * | C | ✓ | [] | ! |
| Ben | Three times four is— | + | R | ✓ | [] | — |
| Eva | —three times four is seven hours. | + | C | X | [] | — |
| Jay | Wrong, three times four is eight hours. | — | C | X | [] | — |
| Ben | If we do three times four, don't we get twelve hours because four plus four plus four is twelve? | — | C | ✓ | J | ? |
| Ana | Yep. | + | N | ✓ | N | — |

^aEvaluation of the previous action (EPA): agreement [+], polite disagreement [—], rude disagreement [—], ignore/new topic[*]. ^bKnowledge content (KC): contribution [C], repetition [R], null academic content [N]. ^cValidity: right [✓], wrong [X], null academic content [N]. ^dJustification: justification [J], no justification [], null academic content [N]. ^eForm of invitation to participate (IF): command [!], question [?], statement [—].

TABLE 3
Summary Table of Hypotheses, Data, Model, and Theories Regarding Correct Contributions (CCs)

| <i>Hypothesis</i> | <i>Data</i> | <i>Model of Predictors</i> | <i>Theory</i> |
|--|---|---|--|
| H-1. Groups with more CCs are more likely to solve the problem correctly. | Final group answers; student characteristics; computed group characteristics; summary statistics of variables coded from 2,951 turns of transcripts | Math grade (mean vs. highest), ratio of CCs over total group turns Other variables: peer status, math grade variance, peer status variance, words, on-task words | Functional group decision making (Orlitzky & Hirokawa, 2001) |
| H-2. CCs occur in more clusters in successful groups. | Final group answers; each group's time periods of high vs. low CCs | <i>t</i> test of differences in time periods in successful vs. unsuccessful groups | Phases vs. reach-tester (Ellis & Fisher, 1994, vs. Pavitt & Johnson, 2001) |
| H-3a. Correct and wrong contributions aid CC creation. | Final answers to algebra problem; student characteristics; group characteristics; variables coded from 2,951 turns of transcripts | CC (-), wrong contribution (-), for $i = 1..4^a$ | Functional group decision making (Orlitzky & Hirokawa, 2001) |
| H-3b. Correct evaluations, questions, and justifications aid CC creation. | Correct evaluations; variables coded from 2,951 turns of transcripts | Correct evaluation (-), question (-), justification (-) for $i = 0..4^a$ | Sociocognitive conflict (Doise et al., 1975; Piaget, 1985) |
| H-3c. Polite disagreements aid CC creation, but rude disagreements, false agreements, and commands hinder. | Polite disagreements aid CC creation, but rude disagreements, false agreements, and commands hinder. | Polite disagree (-), rudely disagree (-), agree (-), command (-) for $i = 1..4^a$ | Politeness (Brown & Levinson, 1987) |
| H-3d. Greater status differences hinder CC creation. | Greater status differences hinder CC creation. | Variances of math grade, peer status, gender, and race ^a | Status characteristics (Cohen, 1982, 1994) |

^aVariables in full model. Classroom identification variables: Class_1, Class_2, Class_3. Group-level variables: correct solution, mean math grade, mean peer status, math grade variance, peer status variance, gender variance, race variance. Current speaker (0) variables: gender, race, math grade, peer status, correct evaluation, agree, politely disagree, rudely disagree, justify, question and command. Previous speakers' lag variables ($i = 1..4$): gender (-), race (-), math grade (-), peer status (-), correct evaluation (-), agree (-), politely disagree (-), rudely disagree (-), command (-), wrong contribution (-), correct old idea (-), justify (-), question (-), command (-).

Predicting solution score at the group level. First I ran an analysis of the outcome variable *solution score* to test H-1, that groups with more CCs were more likely to solve the problem. Hierarchical regressions and path analyses were used to test for total, direct, and indirect effects on solution score. Solution score was an ordered variable, not a continuous one, so using a least squares regression would have biased the estimation of the standard errors (Kennedy, 2004). Ordered logit addressed this problem (Kennedy, 2004).

To predict solution score, I added the following independent variables to the regression. First classroom identification binary variables were entered to control for classroom effects. Then the group's mean mathematics grade and its members' highest mathematics grade were entered into the regression both separately and together. If both were significant predictors alone but neither was significant together, the one that explained more solution variance (McFadden's, 1974, R^2) was kept. The variables *total number of words* and *total number of on-task words* controlled for the total talk in each group. Time constrains the direction of causality, so group processes cannot affect characteristics prior to the group problem solving. Hence, I entered characteristics of group members into the regression before entering group processes. The order was as follows: mathematics grade (mean and/or highest), peer status, mathematics grade variance, peer status variance, words, on-task words, and percentage of CCs over total group turns. (Unlike percentage of CCs over total turns, a simple CC total favors groups that generate many ideas, both correct and incorrect. Meanwhile, the ratio of CCs over new ideas is a measure of accuracy that might overrate groups that produce few ideas. Regressions of solution scores with these other variables tested the robustness of the results.)

A nested hypothesis test (chi-square log likelihood) checked whether each set of added variables was significant (Kennedy, 2004). Only significant variables were retained in subsequent regressions.

A path analysis tested for direct and indirect effects. As time constrains the direction of causality, the predictors were entered in temporal order into the path analysis. These computations were performed with the statistical software EVIEWS (Lilien, Startz, Ellsworth, Noh, & Engle, 1995). As the underlying distribution was not known, I repeated these analyses with ordered probit to ensure that the results did not depend on the logit distribution. Note that the small sample size ($N = 20$) limited the statistical power of this analysis to identify nonsignificant results at the group level (power = 0.25 for an effect size of 0.3).

Addressing difficulties of group process analyses. Statistical analyses of group processes at the speaker turn level must overcome three difficulties. First, group members' behaviors and effects differ across groups and across time (nested data). Second, the outcome variable is discrete, not continuous. Third, events are often similar to recent events in time-series data (serial correlation).

Ordinary least squares regressions do not address these difficulties. First, ordinary least squares often underestimates the standard errors of regression coefficients when applied to nested data (Goldstein, 1995). Second, ordinary least squares is inefficient for discrete variables and yields biased results (Kennedy, 2004). Lastly, if the time-series relationships are not modeled properly, the model residuals can be serially correlated, resulting in inefficient parameter estimates and biased estimates of the parameters' standard errors (Kennedy, 2004).

Thus, I addressed these difficulties by using a statistical discourse analysis tool, DMA (Chiu & Khoo, 2005). DMA identifies distinct time periods, tests for group and time period differences, builds an explanatory model for CCs, tests for serial correlation, and models direct and indirect effects. See Appendix C for the underlying mathematics equations.

Watersheds separate distinct time periods of many versus few CCs. Within a problem-solving session, there might be fewer CCs at the start when people are trying to understand the problem than at the end when they are close to a solution. Hence, dividing the time-series data into time periods with significantly more versus fewer CCs allowed for testing of H-2 (groups with more clusters of CCs are more likely to solve the problem correctly) and testing of predictors' different effects across time.

For each group, I used a modified version of the method outlined in Maddala and Kim (1998) based on information criteria to identify the watersheds in time (breakpoints) that divided each group's problem-solving activity into distinct time periods. Conceptually, information criteria measure whether a model strikes a good balance between parsimony and goodness of fit. Unlike other information criteria, the Schwarz or Bayesian information criterion (BIC) provides a consistent estimator for the number of lagged variables in the true model (Grasa, 1989). Predicting the outcome variable, CC, I added locations of possible breakpoints as independent variables and computed the BIC for a simple univariate time-series model (an autoregressive order 1 model). Assuming a given number of breakpoints (first 0 breaks, then 1 break, then 2 breaks, etc.) and using the model above, I calculated the BIC for all possible locations of those breakpoints in the time series. (For example, for one break, I calculated the BIC if the break is between Turn 1 and Turn 2, then if it is between Turn 2 and Turn 3, etc.) This was done for all possible numbers of breakpoints from 0 to 5. (Current microcomputers lack the computational speed to test more than five breakpoints [six time periods]). The optimal model has the lowest BIC. Applying this method to each group yielded the number and locations of breakpoints (and hence time periods) for each group.

Then I used a *t* test to determine whether successful groups have more CC clusters/time periods than unsuccessful groups (H-2). The small sample size limited the statistical power of this analysis to identify nonsignificant results (number of

time periods = 72; power = 0.75 for an effect size of 0.3). Transcript segments illustrate representative breakpoints.

Predicting CCs at the speaker turn level. I used a multilevel logit variance components model (Bryk & Raudenbush, 1992; Goldstein, 1995) to test if the outcome variable, CC, significantly varied across groups or across time periods. Multilevel models separated unexplained error into speaker turn (Level 1), time period (Level 2), and group (Level 3) variance components, thereby removing the correlation among error terms resulting from speaker turns nested within time periods within groups. If the variance components model showed significant variation at both the group and time period levels, then both the groups and time periods were heterogeneous. In that case, a three-level model was needed.

Next I entered the following independent variables. First I added a vector of s classroom identification variables as control variables (**S**). As the likelihood ratio test for significance of additional explanatory variables was not reliable for this estimation method, Wald tests were used (Goldstein, 1995). Nonsignificant variables were removed from the specification.

Then I entered t variables at the group level: correct group solution, mean of group members' mathematics grades, mean of group members' peer statuses, variance of mathematics grades, and variance of peer statuses (**T**). The last two variables tested status effects (H-3d). As with **S**, a Wald test was done on **T**. Then I tested for interaction effects among pairs of significant variables in **T**. Nonsignificant variables and interactions were removed from the specification.

Next I added u current speaker variables at the speaker turn level: gender, race, mathematics grade, peer status, correct evaluation, agree, politely disagree, rudely disagree, justify, question, and command (**U**). Likewise, I applied the procedure for **T** to **U**. Then I tested if the speaker turn level regression coefficients differed significantly at the time period or group levels (Goldstein, 1995). If yes, I kept these parameters in the model. Otherwise, I removed them.

Using a vector autoregression (Kennedy, 2004), I entered lag variables for the previous speakers, first at lag 1 (indicating the previous turn and denoted -1), then at lag 2 (denoted -2), then at lag 3, and so on until none of the variables in the last lag were significant (lag 4 in this case). First I added v previous speaker variables: gender (-1), race (-1), mathematics grade (-1), peer status (-1), correct evaluation (-1), agree (-1), politely disagree (-1), rudely disagree (-1), CC (-1), wrong contribution (-1), correct old idea (-1), justify (-1), question (-1), and command (-1) (**V**). As shown in Figure 1, these variables tested the new ideas, argumentation, and rudeness hypotheses (3a, 3b, and 3c). I applied the procedure for **U** to **V**. Then I repeated the procedure for lags -2 , -3 , and -4 of the variables in **V**. The parameters were estimated first with marginal quasi-likelihood, and these results served as starting values for predictive quasi-likelihood estimation (Goldstein, 1995).

All statistical tests used an alpha level of .05. Benjamini, Krieger, and Yekutieli's (2006) two-stage linear step-up procedure controlled the false discovery rate, as computer simulations showed that their procedure addressed this issue better than 13 other methods.

I used Ljung–Box (1979) Q statistics to test for serial correlation (up to order 4) in the residuals for all 20 groups. If the residuals are serially correlated, then the parameter estimates are likely inefficient and standard error estimates are likely biased (Kennedy, 2004). Then the explanatory model must be modified with extra lagged outcome variables (lags of CC) or direct modeling of the serial correlation (see Goldstein, 1995, for details).

Based on the multilevel analysis results, the path analysis estimated the direct and indirect effects of the significant predictors separately to compute their total effects (Kennedy, 2004). As time constrains the direction of causality, I entered the explanatory variables in temporal order into the path analysis.

To aid the interpretation of these results, I converted each predictor's total effect (direct plus indirect) to odds ratios, reported as the percentage increase or decrease (+X% or -X%) in a CC's likelihood (Kennedy, 2004). I repeated these above analyses with multilevel probit to test if the results depended on the logit distribution. I also estimated the predictive accuracy of the final model by comparing the final model's prediction of whether a CC occurred at each speaker turn in each group (y_{ijk}^*) with the CC's actual presence or absence (y_{ijk}).

A multilevel analysis has multiple units of analysis, so the statistical power for each one (group, time period, speaker turn) must be computed separately. As noted earlier, the statistical power for groups and time periods were fairly low, so nonsignificant results at these levels must be interpreted cautiously. At the speaker turn level, however, the sample size was 2,951, so the statistical power was more than 0.99 even for a small effect size of 0.1. None of these units of analyses (speaker turn, time period, group, classroom, school, country) are necessarily representative, so results might differ in other contexts. As students can change behaviors during another student's speaking turn, modeling students as a level of analysis requires multivariate outcome, multilevel, cross-classification logit/probit, but no implementation of such a method has been shown at the publication time of this journal article.

RESULTS

After reporting the preliminary results, I show that groups with a larger percentage of CCs had higher solution scores. Then I examine the differences across time periods, followed by the predictors of CCs at the speaker turn level. Due to space considerations, I include only the main results here; all other results are available upon request.

Preliminary Results

Of the 3,153 total speaker turns, 49 turns were not coded because of poor sound quality (see Appendix D, Table D1). The omitted turns did not significantly correlate with other variables, so they likely did not affect the results. As lag variables required data from preceding turns, 153 turns could not be used. Coding of each dimension showed high interrater reliability (see Appendix D, Table D2).

The summary statistics showed that CCs occurred only 20% of the time (see Table 1), more often in successful groups that solved the problem (26%) than in unsuccessful ones (16%; see Table 4). Moreover, successful groups often had higher mathematics grades than unsuccessful groups (91 vs. 82). Compared to unsuccessful groups, successful groups were more likely to evaluate ideas correctly and justify their ideas and were less likely to disagree rudely (38% vs. 24%, 18% vs. 13%, and 6% vs. 12%, respectively for successful and unsuccessful groups).

TABLE 4
Summary Table of Speaker Turn Variables for Successful and Unsuccessful Groups

| Variable | Overall | | Successful | | Unsuccessful | | Min | Max |
|------------------------|---------|------|------------|------|--------------|------|-----|-----|
| | M | SD | M | SD | M | SD | | |
| Unsolved ^a | 0.50 | 0.50 | 0 | 0 | 1 | 0 | 0 | 1 |
| Correct contribution | 0.20 | 0.40 | 0.26 | 0.44 | 0.16 | 0.37 | 0 | 1 |
| Before problem solving | | | | | | | | |
| Girl | 0.47 | 0.50 | 0.42 | 0.49 | 0.50 | 0.50 | 0 | 1 |
| Asian | 0.15 | 0.35 | 0.13 | 0.34 | 0.16 | 0.36 | 0 | 1 |
| Latino | 0.25 | 0.43 | 0.31 | 0.46 | 0.21 | 0.41 | 0 | 1 |
| White | 0.26 | 0.44 | 0.20 | 0.40 | 0.30 | 0.46 | 0 | 1 |
| Mathematics grade | 86 | 10 | 91 | 8 | 82 | 10 | 64 | 99 |
| Peer status | 6 | 6 | 8 | 7 | 6 | 5 | 2 | 9 |
| During problem solving | | | | | | | | |
| New ideas | | | | | | | | |
| Wrong contribution | 0.10 | 0.30 | 0.10 | 0.31 | 0.09 | 0.29 | 0 | 1 |
| Argumentation | | | | | | | | |
| Correct evaluation | 0.30 | 0.46 | 0.38 | 0.49 | 0.24 | 0.43 | 0 | 1 |
| Polite disagreement | 0.18 | 0.39 | 0.19 | 0.39 | 0.18 | 0.39 | 0 | 1 |
| Ignore/unresponsive | 0.17 | 0.37 | 0.12 | 0.33 | 0.20 | 0.40 | 0 | 1 |
| Question | 0.24 | 0.43 | 0.27 | 0.44 | 0.22 | 0.42 | 0 | 1 |
| Justification | 0.15 | 0.36 | 0.18 | 0.38 | 0.13 | 0.33 | 0 | 1 |
| Face and rudeness | | | | | | | | |
| Rude disagreement | 0.10 | 0.29 | 0.06 | 0.23 | 0.12 | 0.33 | 0 | 1 |
| Agreement | 0.56 | 0.50 | 0.63 | 0.48 | 0.50 | 0.50 | 0 | 1 |
| Command | 0.07 | 0.26 | 0.10 | 0.30 | 0.05 | 0.22 | 0 | 1 |

^aSeparate analyses for groups with each solution score showed substantial differences between groups that did and did not solve the problem correctly, and similar results across the latter unsuccessful groups. Thus, *unsolved* was coded as a binary variable (0 or 1) in the turn-level analysis to facilitate interpretation of the results.

Predicting Solution Score at the Group Level

As expected, the students found the problem difficult. Only 10 of the 20 groups solved it correctly, and every group made at least three mistakes. See Table 1 for overall summary statistics, Table 3 for summary statistics of successful groups that solved the problem versus unsuccessful groups, and Table D3 (Appendix D) for the correlation matrix. All groups were reach-testers; no group used linear problem phases. This result suggests that the natural inclination of these students was to reach-test, similar to many adults (Pavitt & Johnson, 2001; Poole, 1981).

Groups with higher mean mathematics grades or a greater percentage of CCs had higher solution scores, supporting H-1 (see Table 5). When mean mathematics grade and highest mathematics grade were both entered, only mean mathematics grade was significant ($\beta = 0.158$, $SE = .053$, $p < .05$, McFadden's $R^2 = .24$). None of the other predictors were significantly related to solution score. Mean mathematics grade also predicted percentage of CCs ($\beta = .012$, $SE = .005$, $p < .05$, $R^2 = .20$). Replacing percentage of CCs with CC frequency or the ratio of CCs over new ideas yielded similar results but explained less variance.

Watersheds Identify Time Periods of Many Versus Few CCs

CCs did not vary across classrooms or across groups, but CCs varied across time periods and across speaker turns. The classroom identification variables did not significantly predict CCs, so the prevalence of CCs did not differ across classrooms. The variance components model showed that the likelihood of CCs did not vary significantly across groups ($M = 0.000$, $SE = 0.001$), but CCs did vary significantly across time periods ($M = 3.457$, $SE = 0.682$) and speaker turns ($M = 0.908$, $SE = 0.024$). On average, successful groups produced more CCs than unsuccessful groups. However, they did not do so consistently, as CC likelihood differed substantially across time periods within a group. Some time periods had many CCs, whereas other time periods had few CCs. Hence, the likelihood of CCs differed mostly across time periods, not across groups. Of the total variance, less than 0.1%

TABLE 5
Significant, Unstandardized Parameter Coefficients (*SE*) of Hierarchical
Set Ordered Logit Regressions Predicting Solution Score

| Predictor | Regression | |
|-------------------------------|-----------------|-----------------|
| | Model 1 | Model 2 |
| Mathematics grade | 0.256** (0.089) | 0.217* (0.095) |
| Percent correct contributions | | 11.377* (5.033) |
| McFadden's R^2 | 0.228 | 0.377 |

Note: A significant constant term is omitted.

* $p < .05$. ** $p < .01$.

occurred at the group or classroom level, 79% was across time periods, and 21% was within time periods. (The high variance of CCs across time periods [79%] also showed that the breakpoint method effectively identified distinct time periods.) As the variance of CCs across groups was not significant, a two-level model (time periods and turns) with group interaction terms was used.

CCs occurred in similar numbers of clusters in both successful and unsuccessful groups, so the results did not support H-2. (See Figures 2 and 3, which show each group's time periods of high CCs versus low CCs.) The time periods for each group ranged from 1 to 5. The number of time periods did not differ significantly in successful versus unsuccessful groups (successful: $M = 3.3$, $SD = 2.1$; unsuccessful: $M = 3.9$, $SD = 1.7$; t test = 0.702; $p > .05$).

An exploratory analysis of the time periods showed that only three groups had consistent CC production rates. Two groups that consistently produced CCs at a *high rate* exceeding 50% successfully solved the problem. Meanwhile, one group that consistently produced CCs at a *low rate* below 20% failed to solve the problem. Otherwise, CCs clustered in 17 of the 20 groups. Seven of eight groups that started with a high CC rate successfully solved the problem, and all eight groups that ended with a high CC rate successfully solved the problem. Eight of ten groups that ended with low CC rates failed to solve the problem.

As shown in the examples below, an exploratory classification of the 52 breakpoints suggested three broad categories: off-task \leftrightarrow on-task transitions, insights, and critical errors. At 26 of the breakpoints (8 in successful groups, 18 in unsuccessful groups), groups transitioned from off-task to on-task or vice versa. At 14 of the breakpoints (8 in successful groups, 6 in unsuccessful groups), a group member had an insight, and CCs increased sharply. At the remaining 12 breakpoints (7 in successful groups, 5 in unsuccessful groups), group members made a critical error, and CCs fell sharply.

The following transcript examples illustrate the different types of breakpoints that separated time periods of many versus few CCs. At the following breakpoint at Turn 7, the group moved from an off-task conversation to work on the problem. (All names are pseudonyms.)

| Turn | Person | Talk and/or Action |
|------|--------|--|
| 1 | Jim | I'm going to Idaho with my family. |
| 2 | Bob | We went to Maryland last summer. |
| 3 | Jim | I don't want to go there. |
| 4 | Pat | What's the distance between New York and London? |
| 5 | Bob | Oh, thirty-five hundred. We're going; we're going helicopter. |
| 6 | Pat | Something like three thousand five hundred. |
| 7 | Jim | I really think I just don't want to go. Okay, so the boat, how fast is the cruise going? It's twenty-two miles per hour? That's over five miles, so five hours, so five times twenty-two would be, wait. |
| 6 | Tim | [raises hand] Ms. T_____ [teacher's name] |
| 9 | Jim | They're only, they're only really like, luckily, a hundred and ten miles out. |
| 10 | Bob | If they go ninety miles an hour, they can get there in less than two hours. |

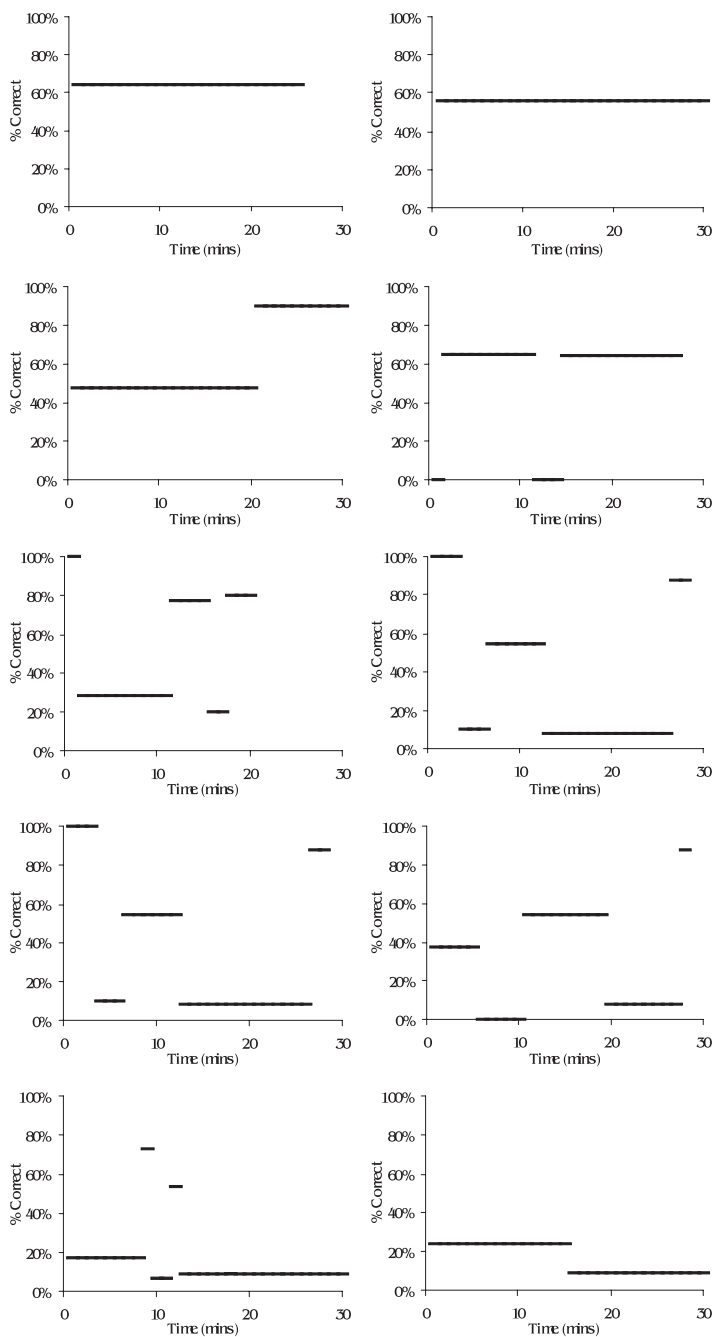


FIGURE 2 Line graphs of the percentage of new ideas that were correct in each time period for groups with correct solutions. Each line segment indicates a distinct time period.

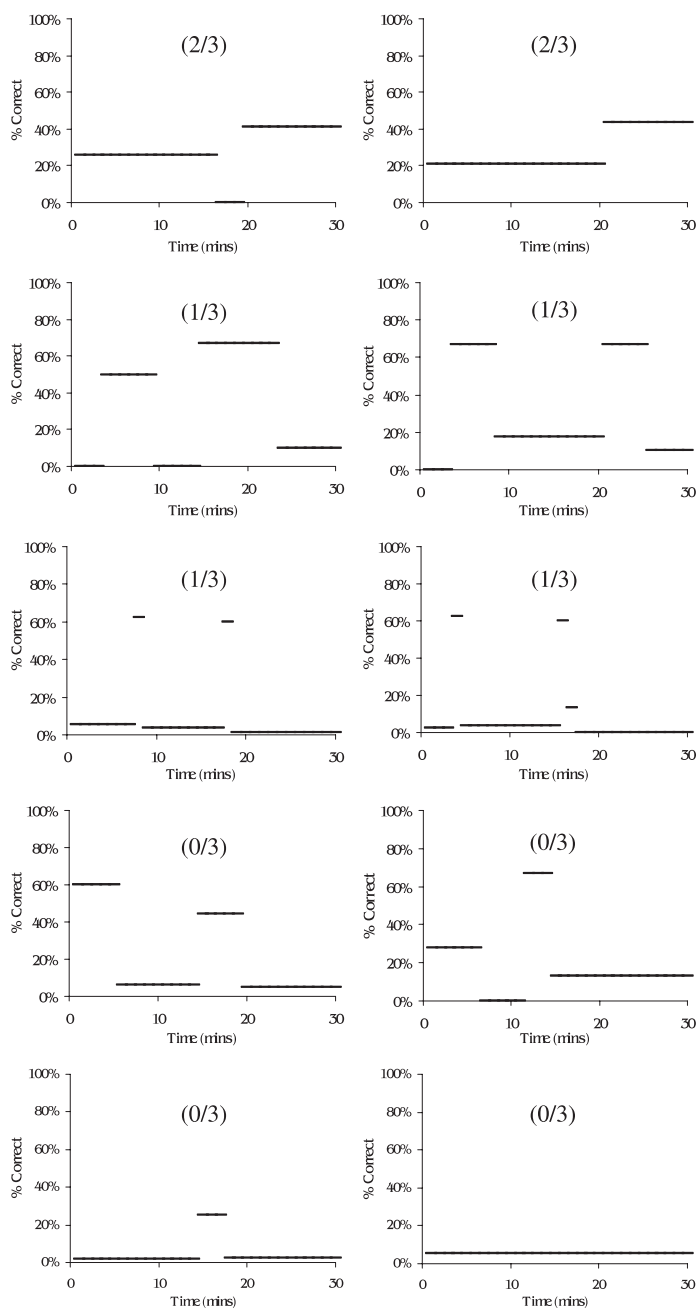


FIGURE 3 Line graphs of the percentage of new ideas that were correct in each time period for groups with incorrect solutions (with scores in parentheses). Each line segment indicates a distinct time period.

Jim and Bob discussed their travels before Pat asked a question about the problem (“What’s the distance between New York and London?”). Although not relevant to the solution, this question drew the students’ attention to the problem. Bob projected himself into the problem situation (“We’re going; we’re going helicopter”) after answering Pat’s question (“Thirty-five hundred”). After Pat acknowledged Bob’s answer, Jim concluded his off-task thoughts (“I really think I just don’t want to go”) and began working on the problem (“Okay, so the boat, how fast ...”). Jim’s proposed multiplication in the breakpoint Turn 7 (“five times twenty-two”) was the first CC in their group. After Tim asked for the teacher, Jim and Bob began a series of CCs (“a hundred and ten miles out” and “they can get there in less than two hours”). So, this breakpoint indicated a change from off-task to on-task behavior at the first CC. (To distinguish on-task time periods from off-task time periods, the breakpoint method can be used on an on-task [vs. off-task] outcome variable to yield an on-task breakpoint at Turn 4.)

Breakpoints also occurred at major insights. In the following example, the students did not make much progress until a student drew a diagram:

- 61 Rex One-ten and ninety is [hits calculator keys, $110 + 90$] two hundred
 62 Amy What’s two hundred?
 63 Rex Two hundred miles?
 64 Liz It’s five hours, so [hits calculator keys, 90×5] Four hundred and fifty?
 65 Amy [draws] Ok, this is like. Okay, so like, ok, this [points to drawing] is New York, right? And that’s London [points to drawing].
 66 Liz Right.
 67 Amy [pointing to drawing] Okay, okay um, that’s the cruise ship. Ok. And like the cruise ship is ahead of the helicopter, right?
 68 Liz Yeah. At a hundred and ten.
 69 Amy Okay, [writes 110 near the cruise ship symbol], the helicopter’s moving up.
 70 Max That’s a helicopter?
 71 Amy Well, [raises open hands] I can’t draw [laughs]
 72 Max [laughs] Alright.
 73 Amy Okay? You’ve got to think about the time. We have ninety–
 74 Liz –We have this um to deal with this [points to cruise ship on drawing] too because it’s not gonna stop.
 75 Amy Oh, the cruise ship’s not gonna stop.

Rex and Liz had been adding, multiplying, and dividing several numbers from the problem (5, 22, 90) without making much progress. After Amy’s questioning of Rex (“What’s two hundred?”) yielded an uncertain answer (“Two hundred miles?”), Amy drew a diagram of the problem situation, correctly modeling the ship’s location, the helicopter’s location, and its movement (a CC in the breakpoint Turn 67). Liz elaborated the diagram with the distance of the cruise ship from shore (“At a hundred and ten”). After some friendly teasing about the quality of the

drawing, Amy highlighted the time traveled by the helicopter. Then Liz interrupted with a CC about the cruise ship's continuing motion ("it's not gonna stop"). Amy validated Liz's idea, and the group then computed each vehicle's movements and marked their new locations on the diagram to solve the problem. In short, the diagram was a breakpoint that ignited several CCs by helping these students model changes in the problem situation rather than simply trying different computations.

Breakpoints also occurred at critical errors. In this example, the group recognized that the helicopter and the ship both moved and tried to compute their movements:

- 91 Bob [hits calculator keys $15 \times 50 = 750$]
 92 Ben What is – what is seventy thousand – seven hundred fifty mean?
 93 Lex I think the wrong things got multiplied; try ninety and one point five.
 94 Bob [laughs, hits calculator keys $1.5 \times 90 = 135$] It's one thirty-five.
 95 Jim I don't know. I'm telling you, you gotta, what you gotta do is divide thirty-three miles into that {helicopter speed?}. Cause twenty-two plus eleven. Thirty-three.
 96 Bob Thirty-three added onto a hundred and ten [writes $33 + 110 =$].
 97 Jim Thirty-three.
 98 Bob Then add.
 99 Jim Why are you adding? No. Just listen. Okay. Ninety. Nine, okay, no, okay, one thirty-three, ninety into one thirty-three. Goes in, um, once. Gives me forty-seven?
 100 Ben [laughs]
 101 Bob [laughs]
 102 Lex [laughs]
 103 Jim Forty-seven? Huh?
 104 Bob Forty-seven?
 105 Jim Yeah. Forty-seven, so. Um, goes in, what?
 106 Bob Try five hours?
 107 Jim Five? Maybe one more.

After Lex noted that Bob hit the wrong calculator keys, Bob laughed and corrected his computation of the helicopter's distance from shore after an hour and a half. At the breakpoint Turn 95, Jim incorrectly disagreed and mistakenly suggested dividing the helicopter speed into the distance traveled by the ship in an hour and a half (33) rather than into the remaining distance between the two vehicles ($= 9 = 143 - 135$; $143 = 33 + 110$). When Bob suggested adding 33 and 110, Jim reasserted his position ("Thirty-three"), challenged Bob ("Why are you adding?"), ignored Bob ("No. Just listen"), changed his mind, and followed Bob's suggestion but added incorrectly ("no, okay, one thirty-three"). Jim compounded this arithmetic error with further division errors to compute $133 \div 90 = 47$ rather than 1.47 (actually, 1.4777). Surprised by the result, Jim did not know how to proceed ("Forty-seven, so. Um,

goes in, what?”). Bob suggested using the 5 hours from the problem, and Jim decided to add one more hour. Neither these ideas nor those for the next few minutes were productive. So, this breakpoint indicated a critical error, changing a productive time period with many CCs to an unproductive time period with few CCs.

Predicting CCs at the Speaker Turn Level

The explanatory model at the speaker turn level showed that wrong, new ideas; argumentation; and politeness affected the likelihood of CCs (see Figure 4 for the path analysis, Table D4 (Appendix D) for the correlation–covariance matrix, and Table D5 for the multilevel logit regression results). However, status differences were not linked to CCs, and CCs did not predict subsequent CCs.

New ideas. These problem-solving sessions had few chain reactions of CCs, as a CC did not help create a subsequent CC. However, a wrong contribution (–1, in the previous turn) was 4% more likely to yield a CC (see Figure 4; +4%: 19% →

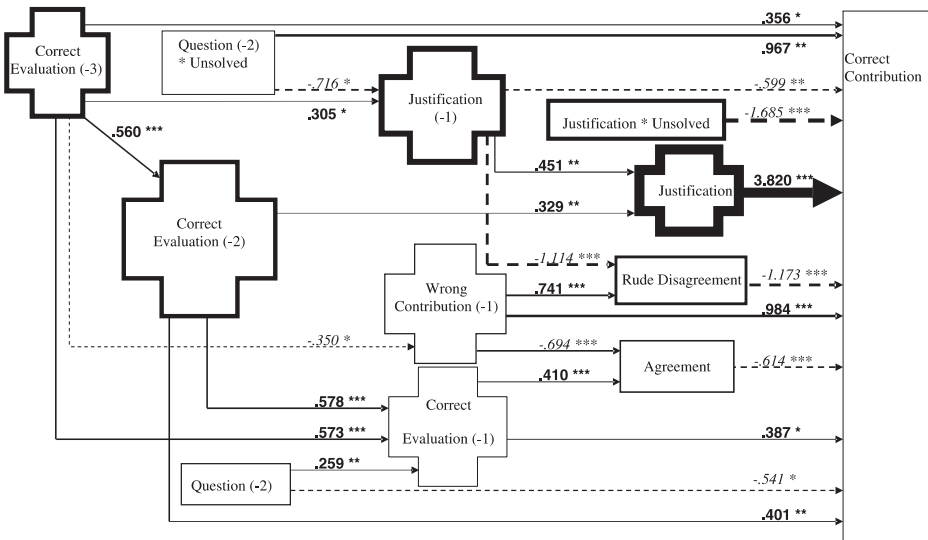


FIGURE 4 Path analysis of significant predictors of correct contributions using multilevel logit. Negative numbers in parentheses (–1, –2, –3) indicate actions that occurred one, two, or three turns ago. Values are standardized parameter coefficients. Crosses (✚) indicate positive overall effects, whereas rectangles (▭) indicate negative overall effects. Solid arrows (→) indicate positive direct effects, whereas dashed arrows (--▶) indicate negative direct effects. Thicker lines indicate larger effects. For example, a correct evaluation in the previous turn (–1) has a direct effect on a correct contribution of +0.387, although its indirect effect is –0.252 (= 0.410 × –0.614), yielding a total effect of +0.135. “Unsolved” refers to speakers in groups that did not correctly solve the problem.

23%; after a turn *without* a wrong, new idea, a CC occurred 19% of the time; after a turn *with* a wrong, new idea, a CC occurred 23% of the time; see Appendix C for computation details). Thus, these results only partially supported H-3a. In response to wrong contributions, group members were more likely to rudely disagree (+7%: 9% → 16%) and less likely to agree (−17%: 57% → 40%), suggesting that they often recognized the flaws of wrong, new ideas. In the following segment, a student incorrectly multiplied the helicopter speed by 5 hours:

Amy: In five hours, multiply [enters 90×5 on calculator] Four hundred and fifty in five hours.

Rex: Four fifty? That can't be right 'cause the cruise ship is only at one-ten. Oh! Oh! The helicopter leaves later! Multiply by two hours! Multiply by two hours!

Rex recognized that the outcome was wrong (“That can't be right”) because the helicopter would have passed the target cruise ship (“is only at one-ten”). This error helped Rex detect and correct the flaw in the number of hours from five to two (“Oh! The helicopter leaves later! Multiply by two hours!”). Building on Amy's partially correct idea to multiply the helicopter speed by 5 hours, Rex created a CC.

Argumentation. Correct evaluations and justifications helped create CCs, but questions did not (partially supporting H-3b). Correct evaluations from any of the three previous speakers (−1, −2, −3) were more likely to yield a CC (+2%, +5%, and +3%, respectively; 19% → 21%; 19% → 24%; 19% → 22%). Each correct evaluation also yielded more subsequent correct evaluations, both in the next turn and in the following turn (+12%: 27% → 39% for both cases). These correct evaluations made a subsequent CC more likely in part through more justifications (+3%: 15% → 18%). Correct evaluations also made wrong contributions less likely (−3%: 10% → 7%) and agreements more likely (+10%: 51% → 61%).

Justifications increased a CC's likelihood in all groups, though more in successful groups (+68%: 14% → 82%) than in unsuccessful groups (+29%: 12% → 41%), possibly because the quality of their justifications differed. In successful groups, members often referred to mathematics relationships to justify their ideas:

Ian: Twenty-two plus five is—

Jo: —we're doing times five 'cause it's rate times time.

When Jo corrected Ian, she referred to the “rate times time” (equals distance) formula.

In unsuccessful groups however, students often justified their claims by citing authority (e.g., teacher, textbook, problem statement):

May: Ninety times five is four fifty.

Kit: Ninety times–

May: – five hours because Ms. T [teacher] said five hours.

Unlike Jo, May justified her computation by incorrectly citing the teacher: “because Ms. T [teacher] said five hours.” Justifications based on mathematics might be more valid, relevant, and helpful to group members than those based on authority, which might explain why justifications had larger effects on CCs in successful groups than in unsuccessful groups. Justifications (–1) also raised the likelihoods of a CC (+36%: 15% → 51%) and a subsequent justification (+4%: 14% → 18%) while reducing that of a rude disagreement (–5%: 11% → 6%).

Questions (–2) reduced a CC’s likelihood (–2%: 21% → 19%). This result suggested that these questions typically indicated individual knowledge gaps. As such, other group members could give an immediate, known explanation, which did not stimulate a CC:

Xiao: Why do we multiply twenty-two times five hours?

Ron: Rate times time is how far the ship moves.

Xiao: Oh! Rate times time. Ok.

When Xiao questioned a computation, Ron justified its validity via the rate–time–distance relationship. Xiao understood (“Oh!”) and accepted it (“Ok”).

In unsuccessful groups however, a question (–2) further reduced the likelihoods of a justification (–1) and of a CC (respectively, –2%: 14.6% → 12.6%; –4%: 17% → 13%). Consider this segment:

Beth: Why ninety times five?

Mark: That’s what the problem said.

Beth: No, it didn’t. It didn’t say do ninety times five.

Mark answered Beth’s question by referring to the problem statement: “That’s what the problem said.” Not satisfied with that answer, Beth challenged Mark with a rude disagreement, “No, it didn’t,” declaring that Mark was wrong as the problem statement did not specify that multiplication (“It didn’t say do ninety times five”). In short, after a question, unsatisfactory responses might help account for CCs being less likely in unsuccessful groups than in successful groups.

Face, rudeness, and status. Polite disagreements increased a CC’s likelihood, supporting H-3b (+14%: 13% → 27%). Consider the transcript segment of Ian and Jo again:

Ian: Twenty-two plus five is–

Jo: –we’re doing times five ‘cause it’s rate times time.

Jo redressed her disagreement by shared positioning (“we”) before justifying her disagreement with Ian’s idea.

In contrast, rude disagreements reduced a CC’s likelihood, also supporting H-3c (−4%: 21% → 17%). Consider the following example:

Eva: Ninety times five, four fifty.

Ada: That’s wrong.

Eva: No, it’s not.

Although Eva’s idea is arithmetically correct, the computation was not consistent with the problem situation and, hence, conceptually incorrect. Ada rudely disagreed with Eva’s computation without explaining: “That’s wrong.” In response, Eva retaliated with a rude disagreement: “No, it’s not.” Thus, rude arguments hindered creation of CCs, whereas polite ones aided their creation.

Controlling for correct evaluations, agreements reduced a CC’s likelihood, also supporting H-3c (−5%: 23% → 18%). These agreements were often simple confirmations such as “yep” or repetitions such as “one ten, right”:

Lana: Uh, ninety times five, four fifty.

Jack: Uh-huh.

Students like Jack often gave brief confirmations (“Uh-huh”) without further elaboration or CCs, suggesting that he might have used false agreements to build social relationships by sacrificing problem-solving progress. Meanwhile, commands did not significantly affect CC creation.

Status and other effects. None of the other predictors were significant. In particular, larger status differences did not affect CCs (no support for H-3d). Neither group mathematics grade nor individual mathematics grade was significant either. Although mean mathematics grade positively correlated with group percentage of CCs in the group-level analysis, that group-level analysis omitted 99.9% of the CC variance.

Only two predictors showed different effects across time periods (agree and correct evaluation [−2]; for detailed regression results, see Appendix D, Table D5). Agreements reduced a CC’s likelihood ($M = -5\%$: 23% → 18%), with the effect varying across time periods from −3% to −21%. Correct evaluation (−2) raised a CC’s likelihood ($M = +5\%$: 19% → 24%), with the effect varying across time periods from −0.3% to +9%. The varying effect sizes of agreement and correct evaluations across time periods suggested that their effects were moderated by unexamined variables that differed across these time contexts. Aside from justifications, agreements, and correct evaluations (−2), the effects of all other predictors did not

differ significantly across time periods or across groups. Hence, the other predictors showed no evidence of contextual effects and are candidates for broader, possibly universal effects.

This model had an 84% accuracy rate for predicting whether a CC occurred in any given turn (y_{ijk}^* vs. y_{ijk}). Furthermore, the Q statistics run on the final model showed no significant serial correlation of residuals in any of the 20 groups. So the time-series model was likely appropriate.

DISCUSSION

Past theoretical models have highlighted the importance of correct, new ideas (CCs) to group problem solving (e.g., Chiu, 2000a, 2001; Hinsz et al., 1997). By understanding how group processes help or hinder CC creation, educators can help students engage in beneficial group processes and avoid harmful ones. This study replicates past research by showing that groups with more CCs are more likely to solve the problem successfully. More important, this study extends this line of research in three ways. First, I showed how satisfactory responses to questions and justifications yielded more CCs in groups that successfully solved the problem. Second, I statistically identified three types of crucial events (breakpoints) that divided each group's problem solving into distinct time periods with more CCs versus fewer CCs. Third, I showed how specific group problem-solving processes in the micro-time context helped create CCs.

Differences Among Successful and Unsuccessful Groups

Groups that successfully solved the problem were more likely to respond to questions with justifications, showed stronger justification effects on CCs, had higher mathematics grades, and had proportionately more CCs. Successful groups often answered their members' questions with satisfactory explanations. Although these explanations did not immediately help create CCs, they helped build partially shared understandings in the problem content space and social solidarity in the social relational space, both of which might have enhanced the micro-time context to help create CCs later. In unsuccessful groups, however, inadequate responses to questions were more often rebuffed with rude disagreements, which in turn hindered creation of CCs.

Justifications showed stronger positive effects on CCs in successful groups, in part because these groups used more justifications that referred to mathematics relationships. In contrast, unsuccessful groups used more authority-based justifications that were less helpful in yielding CCs.

When I examined less than 0.1% of the CC variance at the group level, I found that, in general, groups that had higher mathematics grades or more CCs were

more likely to solve the problem. After including the remaining 99.9% of the CC variance at the speaker turn level, however, group mathematics grades did not significantly affect CC creation. Thus, these results highlight the importance of analyzing group processes at the speaker turn level, not only at the group level. In short, groups that answered one another's questions with explanations or that often used justifications (especially mathematically based ones) created more CCs and were more likely to solve the problem successfully.

Clusters of CCs in Time Periods Separated by Breakpoints

DMA's breakpoint estimation method identified watersheds that altered group problem-solving processes and their effects on CC creation. In all, 85% of these groups did not uniformly create CCs at random intervals throughout their activity. Instead, watersheds divided their problem-solving activity into distinct time periods of many CCs versus few CCs. In contrast to H-2, CCs were clustered in most groups, regardless of the group problem-solving outcome.

An exploratory analysis of the breakpoints between time periods suggested three types of watersheds: off-task \leftrightarrow on-task transitions, insights, and critical errors. In half of the breakpoints, groups switched between primarily on-task versus off-task time periods. During about one quarter of the breakpoints, a group member had an insight that led to many more subsequent CCs. In the remaining breakpoints, a group member made a critical error that sharply reduced the number of subsequent CCs.

Most group processes had similar effects on CC creation across time periods, showing no evidence of context-dependent effects. Only agreement and correct evaluations (-2) had different effects on CCs across time periods, suggesting that one or more unexamined variables might moderate their effects across these different time contexts.

Predicting CCs at the Speaker Turn Level

Group members' recent actions (micro-time context) affected CC creation. Consistent with prior research, correct evaluations, justifications, and disagreements aided CC creation (e.g., Barron, 2003; Cobb, 1995). This study extends this line of research by showing (a) the effects of different types of new ideas, (b) the effect sizes of different aspects of argumentation, (c) the durations of argumentation effects, and (d) the effects of face and rudeness.

New ideas. Group members' wrong contributions aided CC creation, but CCs did not aid creation of subsequent CCs, partially supporting H-3a. After a wrong contribution, group members were more likely to disagree, suggesting that group members often detected and corrected flaws to create a CC. Thus, serving as

grist for CCs outweighed the danger of accepting wrong ideas. In contrast, CCs did not aid subsequent CC creation, possibly because they were not necessarily recognized as correct. Together, these results show that wrong contributions were *more* important than correct ones for creating CCs in this study.

Argumentation. Correct evaluations and justifications both immediately aided CC creation, whereas questions did not, partially supporting H-3b. Correct evaluations had long-lasting effects, helping create CCs over the next three speaker turns. Hence, recognizing ideas as correct or flawed helps group members build on them accordingly (Barron, 2003). Correct evaluations also facilitated subsequent correct evaluations, justifications, and agreements while yielding fewer wrong contributions. Together, these results support the view that correct evaluations help create a valid basis of partially shared understandings in the problem content space for creating CCs.

Justifications had the largest effect on CCs (+68% in successful groups and +29% in unsuccessful groups). Furthermore, the effect of two consecutive justifications on creating CCs was larger than the combined effects of all other predictors. Justifications both elicited further justifications in the problem content space and reduced rude disagreements in the social relational space, thereby facilitating calmer, reason-based discussions that helped create CCs both immediately and in the future.

In contrast, questions yielded fewer CCs, especially in unsuccessful groups. In this study, questions often identified individual knowledge gaps. Thus, other group members could answer these questions with previously discussed ideas, reducing the likelihood of immediately creating a CC. Failing to respond with satisfactory explanations led to more rude disagreements and, eventually, fewer CCs, especially in unsuccessful groups. Hence, answering group members' questions satisfactorily likely built partially shared understandings in the problem content space and solidarity in the social relational space, both of which likely helped create future CCs and a correct problem solution.

Face and rudeness. Polite disagreements created more CCs, but rude disagreements and agreements yielded fewer CCs, supporting H-3c. These results are consistent with the view that polite disagreements reduce interpersonal conflict, aiding understanding of criticisms and fostering CCs. Meanwhile, the results are also consistent with the view that rude disagreements escalate face threats and hinder creation of CCs (Chiu & Khoo, 2003).

Agreements also yielded fewer CCs, suggesting that students had substantial face concerns. Specifically, students' social motives might have inclined them to prefer agreements, sometimes reflexively with simple confirmations (Burgoon et al., 1993; Chiu, 2001). This result is consistent with that of Chiu and Khoo's (2003) study showing that people tend to agree excessively after controlling for the

correctness of the previous speaker's idea. In these cases, students sacrificed progress in the problem content space for progress in the social relational space. Together, the disagreement and agreement results support the view that using *politeness* theory to modify *sociocognitive conflict* theory yields more precise models of group problem-solving processes (Brown & Levinson, 1987; Piaget, 1985).

Implications for Researchers

This study modeled conceptual relationships among group processes affecting the *processes* of creating CCs and applied new methods for analyzing them. Due to the small number of groups, the data are not necessarily representative of group interactions. If future studies show similar findings, these results have the following implications for researchers, teachers, and students.

There are four implications for researchers. First, CC creation differed mostly due to the micro-time context; group and classroom differences accounted for less than 0.1% of the CC variance. To model successful and unsuccessful group processes more precisely, researchers can focus on group members' actions, not only on their group or individual characteristics.

Second, group processes often differed across time. Researchers can model group processes more accurately by analyzing them at various times during an activity. Specifically, watersheds (breakpoints) might radically alter group processes, dividing the activity into distinct time periods. Exploratory analyses of these breakpoints suggests three major categories that future researchers can elaborate on or expand (on-task ↔ off-task transitions, insights, or critical errors). Across time periods, relationships among group processes (or between group processes and outcomes) might remain the same or change substantially.

Third, this study highlights the temporal micro-development of students' interactions (Mercer, 2008), showing how the micro-time context influences group processes. Specifically, it shows how sequences of actions and interactions by the three most recent speakers affect the problem content space, the social relational space, and the creation of CCs. In addition to an activity's broader macro-context, researchers can develop a better understanding of group processes by modeling the micro-time contexts and their effects on group processes.

Lastly, this study showcases a method for systematic, fine-grained analyses of individual or group processes: DMA. Specifically, DMA statistically identifies breakpoints (and their respective time periods) and models individual actions or social interactions over time (Chiu & Khoo, 2005). The breakpoints are watersheds that alter group processes, dividing the session into distinct time periods. Meanwhile, the relational variables across speaker turns, multilevel logit/probit, lag variables, path analyses, and serial correlation tests model social interactions within micro-contexts of time, as well as both group and time period differences. Furthermore, DMA models explanatory variables at the group level, time period

level, and speaker turn level simultaneously. In addition to estimating effect sizes and effect durations of explanatory variables, DMA also identifies differences in effects across groups or across time periods, thereby locating possible moderator effects of unexamined variables at the group or time period levels.

Implications for Teachers and Students

The results suggest that teachers can help create classroom cultural practices to facilitate desirable group interactions. Specifically, teachers can promote a mutually respectful, supportive, accountable, safe, and reflective classroom culture. When students are mutually respectful, supportive, and accountable to one another, they are more likely to answer group members' questions to increase their partially shared understandings. A safe environment reduces students' concerns over loss of face and aids their free expression of new ideas, including wrong ones. Furthermore, an accountable classroom culture facilitates frequent justifications of ideas. As generalizable justifications are more beneficial than appeals to authority, mathematics teachers can help their students develop mathematics norms of discourse so that students can propose, evaluate, and justify their ideas more effectively against mathematics standards of reasoning (Balacheff, 1988; Sfard, 2007). By learning these norms, students can develop mathematics eyes to view the world and acquire and communicate mathematics' structural relationships more easily (Blanton & Kaput, 2003; Franke et al., 2007).

This study also shows that correct evaluations have long-lasting benefits. This result suggests that a supportive, accountable, and reflective classroom culture can help students evaluate one another's ideas carefully without impulsive confirmations or rude rejections. If these results are supported by future studies, these changes in classroom culture might help students realize the potential benefits of cooperative learning.

Limitations and Future Research

This study's limitations include its small sample sizes of higher level units (groups, schools, countries), limited problem content, setting, and methodological assumptions. Due to the small number of time periods, groups, teachers, and classrooms, the data were not necessarily representative of group interactions in classrooms. Likewise, the results might differ across schools or across countries. Furthermore, these students were not accustomed to working together in groups, so students with substantial experience working together might behave differently. Likewise, these results might not apply to students who do not know one another well (e.g., during the first day of class in a new school). These same students might also behave differently during discussions of different mathematics problems (e.g., geometry), let alone problems in other subjects like history. Furthermore,

these students might behave differently in other settings outside of school (e.g., home or playground).

DMA relies on two primary assumptions and requires a minimum sample size. Like other regressions, DMA assumes a linear combination of explanatory variables and independent, identically distributed residuals. (Nonlinear functions can be modeled as individual or multiple explanatory variables; e.g., age^2 .) DMA also has modest sample size requirements. Green (1991) proposed the following heuristic sample size, N , for a multiple regression with M explanatory variables and an expected explained variance R^2 of the outcome variable:

$$N > \{8 \times [(1 - R^2) / R^2] + M\} - 1. \quad (1)$$

For a large model of 20 explanatory variables with a small expected R^2 of 0.10, the required sample size is 91 speaker turns: $= 8 \times (1 - 0.10) / 0.10 + 20 - 1$. Less data are needed for a larger expected R^2 or for smaller models. In practice, two groups of students talking for half an hour will often yield more than 100 speaker turns, sufficient for DMA. Thus, researchers can analyze seemingly “qualitative” data sets through both qualitative and quantitative methods (e.g., both traditional discourse analysis and DMA’s statistical discourse analysis).

In addition to addressing the above limitations, future research can use DMA to model individual or group processes by asking questions of the following form: What affects the likelihood of an action at each moment in time? Consider the following research questions: What sequences of recent teacher or student actions facilitate student use of a specific strategy? What influences development of a student’s science language use (register) over five lessons? How much do these effects differ (if at all) across different people, time periods, artifacts, activities, and so on?

In general, one can apply DMA to predict people’s behaviors with diverse explanatory variables. Researchers can use DMA to examine individual learning/problem solving (e.g., with protocol analysis data; Ericsson, 2001). Or, they can examine verbal and nonverbal interactions among people (students, teachers, parents, computer avatars, therapists, etc.). DMA can be used to predict specific behaviors or their properties (e.g., decisions, gaze, use of metacognitive strategies, vocabulary, etc.). Possible predictors include characteristics of the following: recent events, the time period, the individual, group members, artifacts (e.g., graphs), activity (e.g., discussing a poem), broader contextual factors (e.g., setting), historical factors (e.g., notable school events), or interactions among them.

Careful preparation is needed to perform DMA. Before doing DMA, a researcher should clearly delineate outcome variables, explanatory variables, and units of time (e.g., speaker turn). Furthermore, the sample size of time units should be sufficient (see Equation 1). If coding of variables is needed, the researcher then uses or creates a coding framework to yield sufficiently high interrater reliability

(computed via Krippendorff's alpha). Then the researcher follows the procedure outlined in the Methods section: (a) Estimate breakpoints and time periods, (b) compute the variance components to identify the number of levels, (c) add predictors, (d) test for serial correlation, and (e) estimate direct and indirect effects in a path analysis. For details and further suggestions, see Chiu and Khoo (2005).

DMA can be simplified to omit the breakpoint estimation, extended to include multiple outcomes, or modified to perform a meta-analysis. If the data are naturally divided into time periods, breakpoint estimation might not be needed (though it could confirm the validity of the division of time periods; Chiu & Khoo, 2005). Also, breakpoint estimation requires only specification of the outcome variable and can be done separately without the other DMA components (Chiu & Khoo, 2005). Although the current study tested only one outcome variable, multiple simultaneous outcome variables can also be modeled by adding the outcome variables at the lowest level of the nested data structure (for details, see Goldstein, 1995).

DMA can be used to do meta-analyses of DMA studies to test the generalizability of their results. A DMA meta-analysis combines DMA studies to yield larger samples by adding a "studies" variable at the highest level (Goldstein, 1995). This meta-analysis maintains each DMA study's nested level structure (e.g., turns within time periods within groups) to yield more precise results (unlike current meta-analyses that use only the effect size).

CONCLUSION

This study of 80 high school students working on an algebra problem in groups of four showed that some group processes facilitated creation of correct ideas (CCs). As expected, groups with higher past mathematics grades or proportionately more CCs were more likely to solve the problem correctly. Also, watershed events separated distinct time periods with many versus few CCs.

Recent actions by group members (micro-time contexts) affected the likelihood of a CC at a given moment in time. Specifically, wrong contributions, correct evaluations, justifications, and polite disagreements increased the likelihood of a CC. Students often detected and corrected flaws in wrong contributions to create CCs. Correct evaluations had broad effects, increasing the likelihoods of CCs, justifications, and subsequent correct evaluations. Justifications promoted rational discourse, increasing the likelihood of a subsequent justification and reducing the likelihood of a rude disagreement. Although justifications had the largest effects, correct evaluations had the longest lasting effects (across three speaker turns). In contrast, asking questions, disagreeing rudely, and agreeing reduced the likelihood of a CC.

Some effects differed across groups or across time periods. In groups that successfully solved the problem, justifications had larger effects on CCs and questions were more likely to elicit an explanation. Meanwhile, the effects of agreements and correct evaluations on CCs differed across time periods. Other variables that showed consistent effects across both groups and time periods are candidates for universal effects.

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APPENDIX A PROBLEM, CODING, AND SOLUTIONS

Problem

You won a cruise from New York to London, but you arrive 5 hours late. So, the ship left without you. To catch the ship, you rent a helicopter. The ship travels at 22 miles an hour. The helicopter moves at 90 miles an hour. How long will it take you to catch the ship?

Goal

Find the time at which the ship and the helicopter are in the same location.

Key Problem Situation Understanding

After 5 hours, both vehicles move simultaneously at their respective speeds.

Solution Score

| | |
|--|----------|
| Correct answer | 3 points |
| Articulated at least one of the solution methods below | 2 points |
| Articulated the correct goal and problem situation | 1 point |
| None of the above | 0 points |

Solution 1

Write the distance expression for each vehicle and equate them:

$$\begin{aligned}
 \text{Ship distance} &= \text{Helicopter distance} \\
 \text{Ship speed} \times \text{ship time} &= \text{Helicopter speed} \times \text{helicopter time} \\
 22 \text{ mph} \times (T + 5) \text{ hours} &= 90 \text{ mph} \times T \text{ hours} \\
 22 \times T + 110 - 22 \times T &= 90 \times T - 22 \times T \\
 110 &= 68 \times T
 \end{aligned}$$

$$\begin{array}{rcl} 110 / 68 & = & 68 \times T / 68 \\ 1.6176 & = & T \end{array}$$

Solution 2

Compute current gap between ship and helicopter, distance ship traveled in 5 hours at 22 mph:

$$5 \text{ hours} \times 22 \text{ mph} = 110 \text{ miles}$$

Compute net closing speed, helicopter speed minus ship speed:

$$90 \text{ mph} - 22 \text{ mph} = 68 \text{ mph}$$

Obtain time by dividing current gap by net closing speed:

$$110 \text{ miles} / 68 \text{ mph} = 1.6176 \text{ hours}$$

Solution 3

Estimate the additional time needed by iteratively computing the extra time needed for the helicopter to travel to the ship's momentary new position.

- (a) Compute ship movement after 5 hours
 - (b) Compute helicopter time needed to travel that distance
 - (c) Compute distance ship travels in that time
- Repeat (b) and (c) until the helicopter and the plane are in the same place

| Ship movement | Helicopter movement |
|---|--|
| $22 \text{ mph} \times 5 \text{ hours} =$ | 110 miles |
| | $110 \text{ miles} / 90 \text{ mph} = 1.22222 \text{ hours}$ |
| $22 \text{ mph} \times 1.222 \text{ hours} =$ | 26.8888 miles |
| | $26.8888 \text{ miles} / 90 \text{ mph} = 0.29876 \text{ hours}$ |
| $22 \text{ mph} \times 0.29876 \text{ hours} =$ | 6.57273 miles |
| | $6.57273 \text{ miles} / 90 \text{ mph} = 0.07303 \text{ hours}$ |

And so on ...

$$\text{Time} = 1.22222 + .29876 + .07303 + \dots = 1.6176 \text{ hours}$$

APPENDIX B CODING SPEAKING TURNS

In the case of overlapping speech/nonverbal behaviors, the interrupter's speech/behavior is coded as a separate turn after the interrupted person. Let's say Ron interrupts Ana. If Ana stops talking before Ron stops talking, Ana's turn ends at that point. However, if Ana continues speaking through and after Ron stops speaking, then Ana's speech consists of two turns. The first turn ends at the end of Ron's speech, and the second turn begins after Ron's speech.

If there are multiple interrupters, the interrupters' turns are sequenced according to who spoke first. If multiple people (say Ada and Ben) begin and end at the same time, the speech is coded as follows. Each simultaneous speaker is coded as responding to the previous single speaker.

(a) If the simultaneous speakers say the same thing, the following speakers are coded as responding to all of these simultaneous speakers and the sum of their properties (such as peer status).

(b) If the simultaneous speakers say different things, one of them usually continues arguing his or her position after a brief silent pause. Let's say Ada and Ben both speak, then Ada keeps talking. The turn order would be Ben, then Ada.

(In this data of high school students, a third person never spoke after simultaneous disagreeing speakers stopped. One of the disagreeing speakers always continued.)

Decision Trees for Each Speaker Turn Dimension

Evaluation

Does the speaker respond to the previous speaker?

No, code as *unresponsive/ignore*

Yes, does the speaker fully agree with the previous speaker?

Yes, code as *agree*

No, does the speaker disagree with the previous speaker?

No, code as *neutral*

Yes, does the speaker redress threats to face?

Yes, code as *politely disagree*

No, code as *rudely disagree*

Actions That Redress Threats to Face During Disagreements

Hypothetical (if, let's say)

Indirect responsibility

Passive verbs (get, have)

Passive voice (is multiplied)

Citing other people

First-person plural pronouns (we, our)

(Disagreeing with a question rejects it as not useful; e.g., "You're asking the wrong question")

Knowledge Content, Validity, and Justification

Does the speaker express any mathematics or problem-related information?

No, code as *null content*

Yes, is all of this information on the group's log/trace of problem solving?

Yes, code as *repetition*

No, code as *contribution*

and write specific information in this group's log

Does this information violate any mathematics or problem constraints?

Yes, code as *incorrect*

No, code as *correct*

Does the speaker justify his or her idea?

Yes, code as *justification*

No, code as *no justification*

Invitational Form

Do any of the clauses proscribe an action?

Yes, code as *command*

No, is the subject the addressee?

No, are any of the clauses in the form of a question?

No, code as *statement*

Yes, code as *question*

Yes, is the verb a modal?

No, should the described action have been performed, but not done?

Yes, code as a *command*

No, code as a *question*

Yes, is it a Wh- question (who, what, where, why, when, how)?

Yes, code as a *question*

No, is the action feasible?

Yes, code as a *command*

No, code as a *question*

Example**Invitational Form**

- | | |
|---|----------|
| 1. Can you do it on the calculator, John? (John can use a calculator) | Command |
| 2. Can John do it on the calculator? | Question |
| 3. Can you do it on the calculator, John? (John might not know) | Question |
| 4. I hear someone joking around. (Stop joking) | Command |
| 5. Is someone joking around? (Stop joking) | Command |
| 6. What are you joking about? (Stop joking) | Command |
| 7. What are you joking about? (Joking is ok in class) | Question |
| 8. The board is not erased. | Command |

- | | |
|--|----------|
| 9. Did you erase the board? | Command |
| 10. Did you erase the board? (Speaker can't see the board) | Question |

The *invitational form* decision tree and the accompanying examples are based on Labov (2001) and Tsui (1992).

APPENDIX C MATHEMATICAL EQUATIONS UNDERLYING THE SPEAKER TURN ANALYSES

When Do Correct Contributions (CCs) Occur?

To identify distinct time periods, I created a simple univariate time-series model (an autoregressive order 1 model):

$$y_t = C + \beta y_{t-1} + \varepsilon_t \quad (2)$$

The parameter y_t indicates the value of the outcome variable y at speaker turn t . The parameter y_{t-1} indicates the value of the outcome variable in the previous turn, and β is its regression coefficient indicating its relationship with the outcome variable in the current turn t . Meanwhile, C is a constant and ε_t is the residual at turn t . With breakpoints this model becomes the following:

$$y_t = C + C_2 d_2 + C_3 d_3 + \dots + C_p d_p + \beta y_{t-1} + \varepsilon_t \quad (3)$$

The number of time periods is p , and d_p is the dummy variable associated with time period p . Likewise, C_p is the regression coefficient associated with time period p .

The Bayesian information criterion is defined as follows:

$$- \frac{2}{n} L + \left(\frac{k \ln(n)}{n} \right), \quad (4)$$

where k is the number of estimated parameters, n is the number of observations, and L is the value of the log likelihood function using the k estimated parameters.

Multilevel Logit Model

Conceptually, a multilevel logit model can be divided into its multilevel part and its logit part. Consider a three-level model with an outcome variable y_{ijk} (CC) at speaker turn i of time period j in group k and a logit link function (F):

$$y_{ijk} = \beta_{000} + e_{ijk} + f_{0jk} + g_{00k}, \quad (5)$$

$$\pi_{ijk} = p(y_{ijk} = 1) = F(\beta_{000} + f_{0jk} + g_{00k}) = \frac{1}{1 + e^{-(\beta_{000} + f_{0jk} + g_{00k})}}. \quad (6)$$

The Level 2 variation parameter f_{0jk} represents the deviation of time period j from the overall mean, whereas g_{00k} represents the deviation of group k from the overall mean β_{000} . The probability (π_{ijk}) that an event (e.g., a CC) occurs at turn i of time period j in group k is determined by the expected value of the outcome variable and the logit link function (F). The Level 1 variation e_{ijk} does not contribute to the fixed components and is a random variable only at Level 1. So, I constrained the variance of e_{ijk} to 1 without loss of generality.

Therefore, the observed outcome variable y_{ijk} is as follows:

$$y_{ijk} = \pi_{ijk} + e_{ijk}Z_{ijk}, \quad (7)$$

$$\sigma_e^2 = 1, \quad (8)$$

$$z_{ijk} = [\pi_{ijk}(1 - \pi_{ijk})]^{0.5}. \quad (9)$$

Then I added a vector of s classroom variables as control variables: classroom identification numbers (**S**):

$$\pi_{ijk} = p(y_{ijk} = 1 \mid \mathbf{S}_{00k}, \beta_{00s}) = F(\beta_0 + \beta_{00s}\mathbf{S}_{00k} + f_{0jk} + g_{00k}). \quad (10)$$

I tested whether this set of predictors was significant with a nested hypothesis test (chi-square log likelihood; Kennedy, 2004).

Next I entered a vector of t variables at the group level: correct group solution, mean of group members' mathematics grades, mean of group members' peer statuses, variance of group members' mathematics grades, and variance of group members' peer statuses (**T**):

$$\pi_{ijk} = p(y_{ijk} = 1 \mid \mathbf{S}_{00k}, \beta_{00s}, \mathbf{T}_{00k}, \beta_{00t}) = F(\beta_0 + \beta_{00s}\mathbf{S}_{00k} + \beta_{00t}\mathbf{T}_{00k} + f_{0jk} + g_{00k}). \quad (11)$$

I tested whether this set of predictors was significant with a nested hypothesis test (chi-square log likelihood; Kennedy, 2004). Then I tested for interaction effects among pairs of significant variables in **U**. Nonsignificant variables and interactions were removed from the specification.

Next I added u current speaker variables at the speaker turn level: gender, race, mathematics grade, peer status, correct evaluation, agree, politely disagree, rudely disagree, justify, question, and command (**U**):

$$\pi_{ijk} = F(\beta_0 + \beta_{00s}S_{00k} + \beta_{00r}T_{00k} + \beta_{ujk}U_{ijk} + f_{0jk} + g_{00k}). \quad (12)$$

Likewise, I applied the procedure for **T** to **U**. Next I tested if the *u* speaker turn level regression coefficients ($\beta_{ujk} = \beta_{u00} + f_{ujk} + g_{u0k}$) differed significantly (Goldstein, 1995) at the time period level ($f_{ujk} \neq 0?$) or group level ($g_{u0k} \neq 0?$). If yes, I kept these parameters in the model. Otherwise, I removed them.

Using a vector autoregression (Kennedy, 1994), I entered lag variables for the previous speakers, first at lag 1 (indicating the previous turn and denoted -1), then at lag 2 (denoted -2), then at lag 3, and so on until none of the variables in the last lag were significant (lag 4 in this case). First I added *v* previous speaker variables at the speaker turn level: gender (-1), race (-1), mathematics grade (-1), peer status (-1), correct evaluation (-1), agree (-1), politely disagree (-1), rudely disagree (-1), CC (-1), wrong contribution (-1), correct old idea (-1), justify (-1), question (-1), and command (-1) (**V**):

$$\pi_{ijk} = F(\beta_0 + \beta_{00s}S_{00k} + \beta_{00r}T_{00k} + \beta_{ujk}U_{ijk} + \beta_{vjk}V_{(i-1)jk} + f_{0jk} + g_{00k}). \quad (13)$$

Likewise, I applied the procedure for **U** to **V**. Then I repeated the procedure for lags -2, -3, and -4 of the variables in **V**. Like β_{vjk} , the symbols ϕ_{vjk} , γ_{vjk} , and η_{vjk} denote the regression coefficient matrices for the variables in **V** but at lags -2, -3, and -4 respectively:

$$\pi_{ijk} = F(\beta_0 + \beta_{00s}S_{00k} + \beta_{00r}T_{00k} + \beta_{ujk}U_{ijk} + \beta_{vjk}V_{(i-1)jk} + \phi_{vjk}V_{(i-2)jk} + \gamma_{vjk}V_{(i-3)jk} + \eta_{vjk}V_{(i-4)jk} + f_{0jk} + g_{00k}). \quad (14)$$

APPENDIX D CORRELATION TABLES AND ADDITIONAL ANALYSES

TABLE D1
Summary of Speaker Turns Coded in Successful and Unsuccessful Groups

| <i>Speaker Turn</i> | <i>Group</i> | |
|---------------------|-------------------|---------------------|
| | <i>Successful</i> | <i>Unsuccessful</i> |
| Original | 1,372 | 1,781 |
| Poor sound quality | 21 | 28 |
| Omitted due to lags | 74 | 79 |
| Total used | 1,277 | 1,674 |

TABLE D2
Interrater Reliability of Each Coding Dimension

| <i>Coding Dimension</i> | <i>Agreement (%)</i> | <i>Krippendorff's α</i> |
|-------------------------------|----------------------|---|
| Evaluation of previous action | 96 | .93 |
| Knowledge content | 98 | .98 |
| Correct idea | 99 | .99 |
| Invitational form | 96 | .91 |

TABLE D3
Correlations, Variances, and Covariances of Outcome Variables
and Significant Predictors at the Group Level

| <i>Group-Level Variable</i> | <i>1</i> | <i>2</i> | <i>3</i> |
|----------------------------------|--------------|---------------|--------------|
| 1. Solution score | 1.490 | 5.314 | 0.142 |
| 2. Group mean mathematics grade | 0.658 | 43.715 | 0.519 |
| 3. Percent correct contributions | 0.671 | 0.453 | 0.030 |

Note: The lower left triangle contains the correlations, the bold numbers along the diagonal are the variances, and the upper right triangle contains the covariances.

TABLE D4
Correlations, Variances, and Covariances of Outcome Variables
and Significant Predictors at the Speaker Turn Level

| Speaker Turn Variable | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|-------------------------------|--------------|--------------|--------------|--------------|--------------|--------------|--------------|--------------|--------------|--------------|
| 1. Correct contribution | 0.158 | -0.009 | -0.002 | 0.058 | 0.008 | 0.035 | 0.037 | 0.030 | 0.010 | -0.004 |
| 2. Rude disagreement | -0.079 | 0.087 | -0.054 | -0.009 | -0.009 | -0.004 | -0.006 | -0.003 | 0.004 | -0.005 |
| 3. Agreement | -0.020 | -0.368 | 0.246 | -0.009 | 0.002 | 0.029 | 0.012 | 0.019 | -0.012 | -0.002 |
| 4. Justification | 0.412 | -0.083 | -0.050 | 0.127 | 0.025 | 0.010 | 0.014 | 0.003 | 0.005 | -0.004 |
| 5. Justification (Lag 1) | 0.058 | -0.086 | 0.006 | 0.193 | 0.127 | 0.006 | 0.009 | 0.014 | 0.014 | 0.001 |
| 6. Correct evaluation (Lag 1) | 0.195 | -0.030 | 0.127 | 0.059 | 0.038 | 0.210 | 0.047 | 0.048 | -0.001 | 0.008 |
| 7. Correct evaluation (Lag 2) | 0.211 | -0.046 | 0.055 | 0.093 | 0.060 | 0.230 | 0.208 | 0.047 | 0.004 | -0.016 |
| 8. Correct evaluation (Lag 3) | 0.164 | -0.021 | 0.080 | 0.022 | 0.091 | 0.231 | 0.225 | 0.209 | -0.003 | 0.001 |
| 9. Wrong contribution (Lag 1) | 0.093 | 0.048 | -0.090 | 0.047 | 0.142 | -0.009 | 0.016 | -0.028 | 0.088 | 0.000 |
| 10. Question (Lag 2) | -0.022 | -0.043 | -0.020 | -0.023 | -0.001 | 0.038 | -0.079 | 0.003 | 0.008 | 0.184 |

Note: The lower left triangle contains the correlations, the bold numbers along the diagonal are the variances, and the upper right triangle contains the covariances.

TABLE D5
 Significant, Unstandardized Parameter Coefficients (*SE*) of Hierarchical
 Set Multilevel Logit Regressions Predicting Correct Contributions
 at the Speaker Turn Level

| <i>Predictor</i> | <i>Regression</i> | | | | | |
|--|--------------------|----------------------|----------------------|----------------------|----------------------|----------------------|
| | <i>Model 1</i> | <i>Model 2</i> | <i>Model 3</i> | <i>Model 4</i> | <i>Model 5</i> | <i>Model 6</i> |
| Unsolved | -0.907* (0.463) | -0.581 (0.558) | -0.526 (0.555) | -0.690 (0.552) | -0.647 (0.546) | -0.430 (0.477) |
| Rude disagreement | | -0.970*** (0.256) | -1.177*** (0.260) | -1.198*** (0.265) | -1.238*** (0.265) | -1.173*** (0.260) |
| Agreement | | -0.633*** (0.131) | -0.642*** (0.131) | -0.647*** (0.134) | -0.66*** (0.133) | -0.614*** (0.123) |
| Justification | | 3.725*** (0.276) | 3.827*** (0.276) | 3.803*** (0.277) | 3.849*** (0.276) | 3.820*** (0.284) |
| Justification × Unsolved | | -1.568*** (0.338) | -1.651*** (0.338) | -1.654*** (0.341) | -1.69*** (0.339) | -1.685*** (0.346) |
| Lag 1 predictors | | | | | | |
| Correct evaluation | | | 0.447*** (0.125) | 0.427** (0.128) | 0.402** (0.128) | 0.387* (0.127) |
| Wrong contribution | | | 0.928*** (0.178) | 0.952*** (0.182) | 0.977*** (0.181) | 0.984*** (0.180) |
| Justification | | | -0.550** (0.163) | -0.530** (0.167) | -0.555** (0.166) | -0.599** (0.164) |
| Lag 2 predictors | | | | | | |
| Correct evaluation | | | | 0.414** (0.125) | 0.387** (0.125) | 0.401** (0.108) |
| Question | | | | -0.615** (0.198) | -0.613** (0.197) | -0.541* (0.194) |
| Question × Unsolved | | | | 1.008*** (0.277) | 1.002*** (0.276) | 0.967** (0.272) |
| Lag 3 predictor | | | | | | |
| Correct evaluation | | | | | 0.353* (0.127) | 0.356* (0.126) |
| Random variation of predictors at the time period level | | | | | | |
| Agreement | | | | | | 0.445* (0.149) |
| Correct evaluation | | | | | | 0.555*** (0.135) |

Note: Significant fixed constant term, random time period constant term, and random speaker turn constant terms are omitted.

* $p < .05$. ** $p < .01$. *** $p < .001$.