

VISUALIZING FRACTIONS:

Linking Multiple Representations with Auditory Guidance

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C O N T E N T D O M A I N

Elementary students struggles performing operations with fractions are well documented due to “whole-number” bias

Cognitive tutors that use sound instructional principles can address these issues by providing immediate feedback, and aiding students in making robust connections

A number of studies have shown multiple representations to be effective in aiding robust learning and transfer

(Butcher & Aleven, 2007; Rau, Aleven, & Rummel, 2009)

However, taking advantage of multiple representations presents a challenge for learners to make connections among various types of representations (Ainsworth, 2006)

Some possible ways to address this issue might be:

1. Providing explicit guidance to aid learners in making these connections
2. Providing automatic linking among representations to ease cognitive load
3. Allowing learners to generate solutions that link representations together

HYPOTHESIS

1. Linked representations are more effective than static representations in promoting short and long-term retention and transfer of procedures and concepts associated with adding fractions.

2. Multiple representations combined with explicit verbal guidance are more effective than multiple representations with no guidance in promoting transfer and long-term retention

2a. Linked representations combined with explicit verbal guidance to connect them are more effective than representations without any guidance in promoting short and long-term retention and transfer of procedures and concepts associated with adding fractions

3. Linked representations linked dynamically by a tutor are more effective than representations linked manually by students in promoting short-term retention and transfer of procedures and concepts associated with adding fractions.

3a. Manually linked representations, however, are more effective in promoting long-term retention and transfer

3b. Manually linked representations with guidance are most effective in promoting short and long-term retention and transfer

MATERIALS

fractionAddition-instruction.swf

Fraction Addition Tutor

Solve the fraction addition problem. [Click Me!](#)

Given Fractions **Converted Fractions**

$\frac{1}{4}$ = $\frac{\quad}{\quad}$

$\frac{1}{3}$ = $\frac{\quad}{\quad}$

Simplified Answer

$\frac{\quad}{\quad} = \frac{\quad}{\quad}$ [Done](#)

Note: The interface includes pie charts for visual representation of the fractions and 'Okay' buttons for each step.

fractionAddition-noinstruction.swf

Fraction Addition Tutor

Solve the fraction addition problem.

Given Fractions **Converted Fractions**

$\frac{1}{4}$ = $\frac{\quad}{\quad}$

$\frac{1}{3}$ = $\frac{\quad}{\quad}$

Simplified Answer

$\frac{\quad}{\quad} = \frac{\quad}{\quad}$ [Done](#)

Note: This version of the interface does not include the 'Click Me!' button or the 'Simplified Answer' label.

RESEARCH DESIGN

Prior Knowledge →

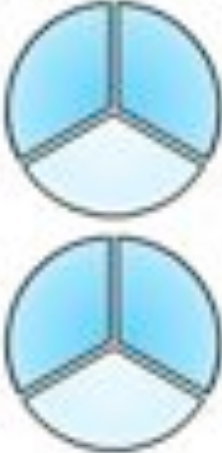
SUPPORT	Multiple Representations			
		Visual + Text	Visual + Text Linked by Student	Visual + Text linked dynamically
	Unguided	A1B1	A1B2	A1B3
	Guided	A2B1	A2B2	A2B3

fractionAddition-noinstruction_self

Fraction Addition Tutor

Solve the fraction addition problem.

Given fractions



Given Fractions

$$\frac{1}{4}$$

+

$$\frac{1}{3}$$

Converted Fractions

$$= \frac{\quad}{\quad}$$

Click Me!

$$= \frac{\quad}{\quad}$$

Simplified Answer

$$= \frac{\quad}{\quad}$$

Done

Hint

Control Condition

- * Static representations
- * No manipulation of visuals or fractions
- * No explicit (or implicit) linking of representations at any level

VISUAL + TEXT

Fraction Addition Tutor

Solve the fraction addition problem. **Cancel**

Given Fractions

Converted Fractions

Given Fractions

$$\frac{1}{4} = \frac{3}{12}$$

Converted Fractions

$$\frac{1}{3} = \frac{4}{12}$$

Simplified Answer

$$\frac{\quad}{\quad} = \frac{\quad}{\quad}$$

Done

Student-Led Condition

- Interactive representations
- Students can manipulate both visual and fractional representations
- Explicit tracing of relationships through sequencing of steps
- Visual mapping of representation relationships through design of interface

Cognitive Tutor Authoring Tools

Start Tools Windows Help

File Mode View Tutor 1

1 Recorder

VISUALLY LINKED TEXT

Fraction Addition Tutor

Solve the fraction addition problem.

Given Fractions

Day

Converted Fractions

Day

+

Given Fractions

 $\frac{1}{4}$

Converted Fractions

Dynamically updated to reflect changing visual representation

+

Given Fractions

 $\frac{1}{3}$

Converted Fractions

Dynamically updated to reflect changing visual representation

Simplified Answer

Learners can manipulate visual representations

G
U
I
D
A
N
C
E

“Notice the relationship between the pictures and the fractions. First, look at the total number of pieces of your pie. You'll notice that corresponds to the denominator in your fraction. Now look at the number of pieces you've selected. That corresponds to the numerator in your fraction. You can represent a fraction with a picture”

TRANSFER TASKS

$$\frac{2}{3} + \frac{1}{5} = ? \longleftarrow \text{NEAR TRANSFER}$$

$$\frac{1}{5} + \frac{3}{4} + \frac{1}{2} = ? \longrightarrow \text{FAR(THER) TRANSFER}$$

$$\frac{2}{7} + \frac{3}{5} = \frac{5}{12} \longrightarrow \text{FAR(THER) TRANSFER}$$

Rob solved the problem above. Is he correct? If not, what error did he make? What should he have done?

Lisa solved the following problems correctly:

$$1 + 9 = 10$$

$$.50 + .25 = .75$$

$$30\% + 20\% = 50\%$$

She tried the same strategy with

$$\frac{1}{3} + \frac{1}{2} = \frac{1}{5}$$

The last problem ^{fr} $\frac{1}{3} + \frac{1}{2} = \frac{1}{5}$. What is Lisa doing wrong?

Why can't she apply the same rules of decimals, percentages, and numbers to fractions?

TRANSFER OF CONCEPT

N E X T S T E P S

1. Given effectiveness of a particular type of multiple representation, investigate whether that principle can work together with other principles (self-explanations, etc...)
2. Investigate whether different types of guidance strategies (auditory vs. visual, or more directed vs. less directed) have different effects on transfer and retention
3. Explore whether results generalize to topics where representations become more complex (linear equations)
4. Investigate whether different multiple representations and guidance have an affect on future learning (converting fractions to decimals, percentages; multiplying and dividing fractions)